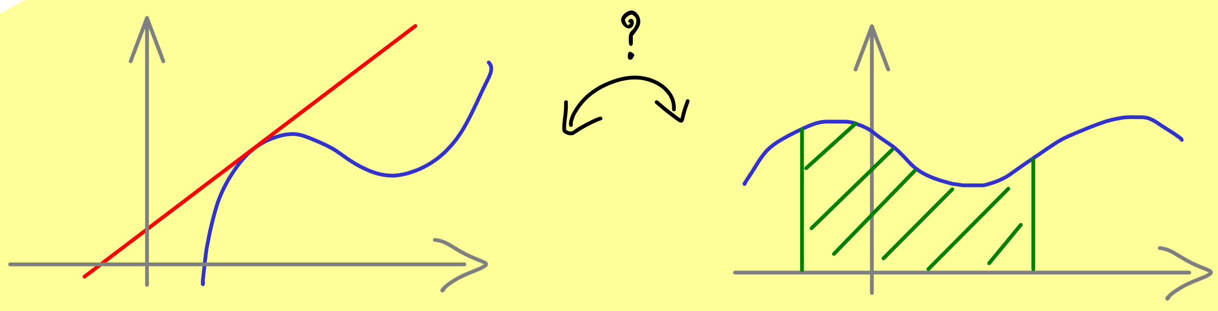
The Bright Side of Mathematics

Real Analysis – Part 54



Definition: Let $I \subseteq \mathbb{R}$ be an interval and $f: I \longrightarrow \mathbb{R}$ be a continuous function. Then a differentiable function $F: I \longrightarrow \mathbb{R}$ is called an antiderivative of f if

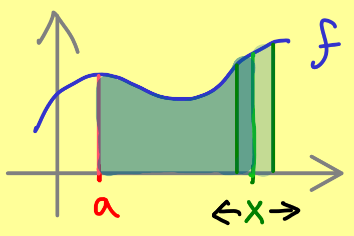
$$F' = f$$

Theorem: (first fundamental theorem of calculus)

I interval, $f: I \longrightarrow \mathbb{R}$ continuous, $a \in I$.

Then $F: I \longrightarrow \mathbb{R}$ defined by $F(x) := \int_a^x f(t)\, dt$

is differentiable and an antiderivative of f: $F' = f$

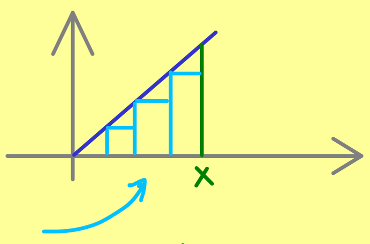


Examples: (a) $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2 \quad \Rightarrow \quad F(x) = \frac{1}{3}x^3$ is an antiderivative

$F_1(x) = \frac{1}{3}x^3 + 1$ is an antiderivative

for $c \in \mathbb{R}$: $F_c(x) = \frac{1}{3}x^3 + c$ is an antiderivative

(b) $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x$, $a = 0$



width: $\frac{x}{n}$, height: $k \cdot \frac{x}{n}$

$$\sum_{k=0}^{n-1} \left(\frac{x}{n}\right) \cdot \left(k \cdot \frac{x}{n}\right) = \frac{x^2}{n^2} \sum_{k=0}^{n-1} k$$

$$= \frac{x^2}{n^2} \frac{(n-1)\cdot n}{2} = \frac{x^2}{2} \cdot \left(1 - \frac{1}{n}\right) \xrightarrow{n \to \infty} \frac{x^2}{2} = \int_0^x f(t)\, dt$$