

The Bright Side of Mathematics

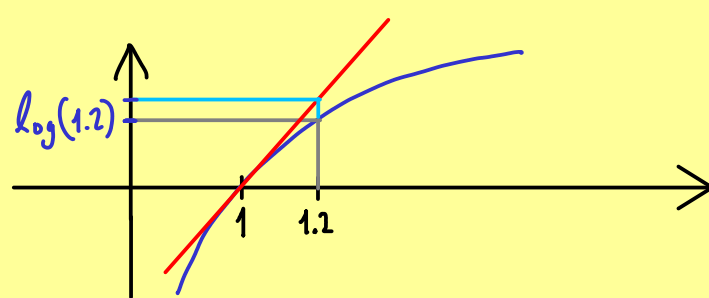
Real Analysis - Part 46

Taylor: $f(x_0 + h) = T_n(h) + R_n(h)$

$$\sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!} \cdot h^k = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot h^{n+1}$$

n -th order Taylor polynomial ξ between x_0 and $x_0 + h$

Example: $\log(1.2) = ?$



expansion point $x_0 = 1$
 $h = 0.2$

$\log(x)$	$\log'(x) = \frac{1}{x}$	$\log''(x) = -\frac{1}{x^2}$	$\log'''(x) = \frac{2}{x^3}$	$\log^{(4)}(x) = -\frac{3!}{x^4}$
$\log(x_0) = 0$	$\log'(x_0) = 1$	$\log''(x_0) = -1$	$\log'''(x_0) = 2$	

Third order Taylor polynomial: $T_3(h) = 0 \cdot h^0 + \frac{1}{1!} h^1 + \frac{-1}{2!} h^2 + \frac{2}{3!} h^3$

$$= h - \frac{1}{2} h^2 + \frac{1}{3} h^3$$

$$T_3(0.2) = \frac{1}{5} - \frac{1}{2} \left(\frac{1}{5}\right)^2 + \frac{1}{3} \left(\frac{1}{5}\right)^3 = \frac{137}{750} = 0.182\bar{6}$$

first digits of $\log(1.2)$?

$$\left| \log(1.2) - T_3(0.2) \right| = \left| R_3(0.2) \right| = \left| \frac{f^{(3+1)}(\xi)}{(3+1)!} \cdot 0.2^{3+1} \right|$$

$$= \left| -\frac{3!}{\xi^4} \cdot \frac{1}{4!} \cdot 0.2^4 \right| = 4 \cdot 10^{-4} \cdot \frac{1}{\xi^4} \leq 0.0004$$

$\xi \in (1, 1.2)$

$$0.182\bar{6} - 0.0004 \leq \log(1.2) \leq 0.182\bar{6} + 0.0004$$

$$0.1822 \leq \log(1.2) \leq 0.1831 \quad \Rightarrow \quad \log(1.2) = 0.18\dots$$