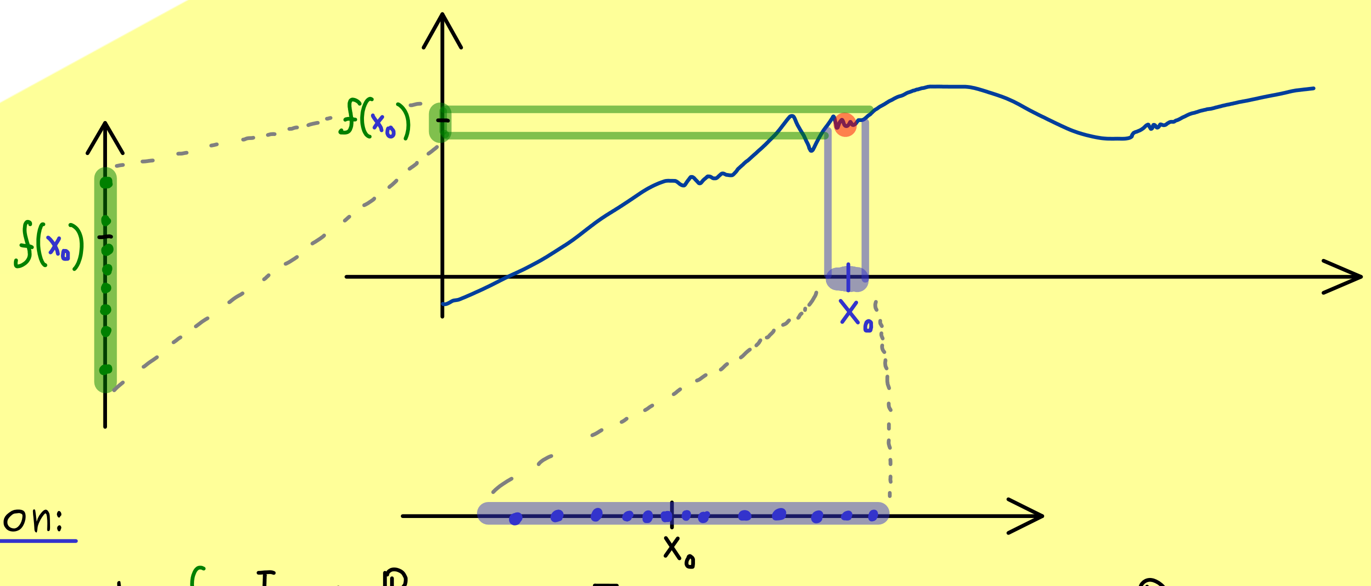
The Bright Side of Mathematics

Real Analysis – Part 26



Definition:

Let $f: I \to \mathbb{R}$, $x_0 \in I$. If there is $c \in \mathbb{R}$ and

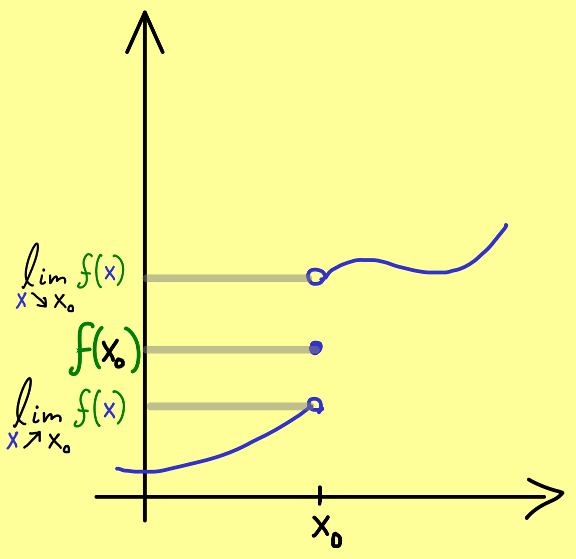
<u>all</u> sequences $(x_n)_{n\in\mathbb{N}} \subseteq I \setminus \{x_0\}$ with $\lim\limits_{n\to\infty} x_n = x_0$

we have $\big(f(x_n)\big)_{n\in\mathbb{N}}$ is also convergent with $\lim\limits_{n\to\infty} f(x_n) = c$,

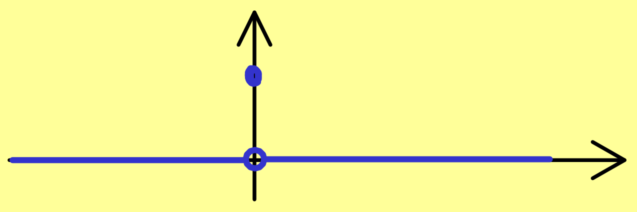
then we write $\boxed{\lim\limits_{x\to x_0} f(x) = c}$

and $\boxed{\lim\limits_{x\nearrow x_0} f(x) = c}$ if $x_n < x_0$ for all $n\in\mathbb{N}$

and $\boxed{\lim\limits_{x\searrow x_0} f(x) = c}$ if $x_n > x_0$ for all $n\in\mathbb{N}$



Example: (a) $f(x) = \begin{cases} 0 & , x \neq 0 \\ 1 & , x = 0 \end{cases}$



$\lim\limits_{x\to 0} f(x) = 0 \neq 1 = f(0)$

(b) $f(x) = a_m \cdot x^m + a_{m-1} x^{m-1} + \dots + a_1 \cdot x^1 + a_0$ $\quad (f: \mathbb{R} \to \mathbb{R})$

For $x_0 \in \mathbb{R}$ take $(x_n)_{n\in\mathbb{N}}$ with $\lim\limits_{n\to\infty} x_n = x_0$

$f(x_n) = a_m \cdot x_n^m + a_{m-1} x_n^{m-1} + \dots + a_1 \cdot x_n^1 + a_0$

$\xrightarrow[\text{(limit theorems)}]{n\to\infty} a_m \cdot x_0^m + a_{m-1} x_0^{m-1} + \dots + a_1 \cdot x_0^1 + a_0 = f(x_0)$

$\Rightarrow \lim\limits_{x\to x_0} f(x) = f(x_0)$