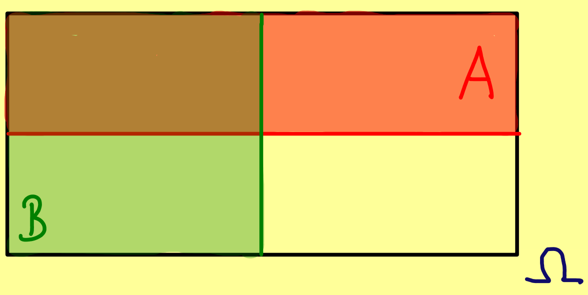
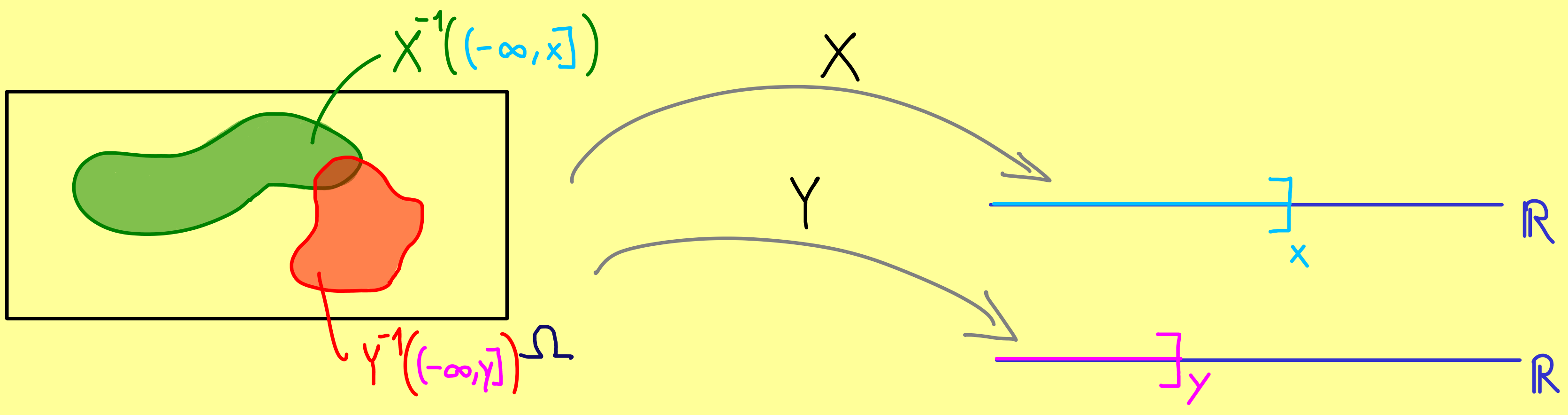
The Bright Side of Mathematics

Probability Theory – Part 13



$A, B \subseteq \Omega$ two independents events

$X : \Omega \longrightarrow \mathbb{R}$, $Y : \Omega \longrightarrow \mathbb{R}$ two independent random variables?



Definition: Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and let $X : \Omega \longrightarrow \mathbb{R}$, $Y : \Omega \longrightarrow \mathbb{R}$ be two random variables.

Then X, Y are called independent if for all $x, y \in \mathbb{R}$

$X^{-1}((-\infty, x])$ and $Y^{-1}((-\infty, y])$ are independent events.

$$\Longleftrightarrow \quad \mathbb{P}\big(X^{-1}((-\infty,x]) \cap Y^{-1}((-\infty,y])\big) = \mathbb{P}\big(X^{-1}((-\infty,x])\big) \cdot \mathbb{P}\big(Y^{-1}((-\infty,y])\big)$$

$$\Longleftrightarrow \quad \underbrace{\mathbb{P}(X \leq x, Y \leq y)}_{F_{(X,Y)}(x,y)} = F_X(x) \cdot F_Y(y)$$

$F_{(X,Y)}(x,y) \longleftarrow$ cdf of random variable $(X,Y) : \Omega \longrightarrow \mathbb{R}^2$

Example: Product space: $\Omega = \Omega_1 \times \Omega_2$, $X : \Omega \longrightarrow \mathbb{R}$, $X(\omega_1, \omega_2) = f(\omega_1)$

$Y : \Omega \longrightarrow \mathbb{R}$, $Y(\omega_1, \omega_2) = g(\omega_2)$

$\Longrightarrow$ X, Y are independent random variables

Definition: A family $(X_i)_{i \in I}$ is called independent if

$$\mathbb{P}\big((X_j \leq x_j)_{j \in J}\big) = \prod_{j \in J} \mathbb{P}(X_j \leq x_j)$$

for all $x_j \in \mathbb{R}$

for all finite $\emptyset \neq J \subseteq I$