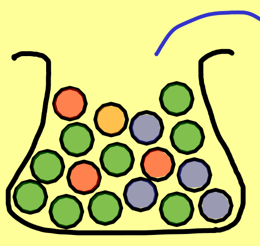
The Bright Side of Mathematics

Probability Theory – Part 6

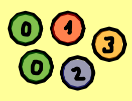
Hypergeometric distribution (multivariant)

size n , unordered , without replacement

urn model  draw n balls at once

colours: finite set $\mathcal{C}$

(for example: $\mathcal{C} = \{0,1,2,3\}$)

(one possible outcome:  $n=5$)

function $\mathcal{C} \to \mathbb{N}_0$ or $(2,1,1,1)$

Sample space:

$$\Omega = \Big\{ (k_c)_{c\in\mathcal{C}} \in \mathbb{N}_0^{\mathcal{C}} \;\Big|\; \sum_{c\in\mathcal{C}} k_c = n \Big\}$$

For our example: $\Omega = \big\{ (k_0, k_1, k_2, k_3) \in \mathbb{N}_0^4 \;\big|\; k_0 + k_1 + k_2 + k_3 = n \big\}$

N_c = number of balls for colour c in the urn

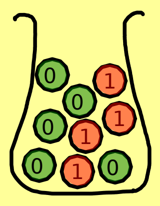
$N := \sum_{c\in\mathcal{C}} N_c$ total number of balls

$$\mathbb{P}(\{(k_0, k_1, k_2, k_3)\}) = \frac{\binom{N_0}{k_0}\cdot\binom{N_1}{k_1}\cdot\binom{N_2}{k_2}\cdot\binom{N_3}{k_3}}{\binom{N}{n}}$$

(multivariant) hypergeometric distribution: $\mathbb{P}(\{(k_c)_{c\in\mathcal{C}}\}) = \dfrac{\prod_{c\in\mathcal{C}} \binom{N_c}{k_c}}{\binom{N}{n}}$

Hypergeometric distribution for two colours:

$\mathcal{C} = \{0,1\}$, $N_0 + N_1 = N$



count the ①s : $\Omega = \{0,1,2,\dots,n\}$

$$\mathbb{P} : \mathcal{P}(\Omega) \to [0,1] \;,\quad \mathbb{P}(\{k\}) = \frac{\binom{N_1}{k}\cdot\binom{N-N_1}{n-k}}{\binom{N}{n}}$$