



# The Bright Side of Mathematics

## Jordan normal form - part 3

$$A = \begin{pmatrix} 2 & 1 & -1 & 8 & -3 \\ 0 & 2 & 0 & 7 & 5 \\ 0 & 0 & 2 & 7 & 5 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Recipe: (1) Eigenvalues  
(2) Alg. and geom. multiplicities  
(3) Dimensions of the generalised eigenspaces

Find a Jordan normal form for A:

(1) Eigenvalues:  $\det(A - \lambda \mathbb{1}) = (2 - \lambda)^5 \Rightarrow \lambda_1 = 2$

For  $\lambda_1 = 2$ : (2) Multiplicities:  $\alpha(\lambda_1) = 5$

$$\text{Ker}(A - \lambda_1 \mathbb{1}) = \text{Ker} \begin{pmatrix} 0 & 1 & -1 & 8 & -3 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{III}-\text{II}} \text{Ker} \begin{pmatrix} 0 & 1 & -1 & 8 & -3 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow \gamma(\lambda_1) = 3$  (three boxes!)

Possibility one:

$$\begin{pmatrix} \boxed{2} & 1 & & & \\ & \boxed{2} & & & \\ & & \boxed{2} & & \\ & & & \boxed{2} & \\ & & & & \boxed{2} \end{pmatrix}$$

Possibility two:

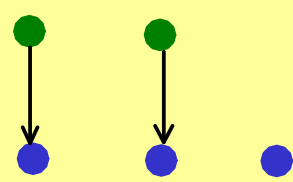
$$\begin{pmatrix} \boxed{2} & 1 & & & \\ & \boxed{2} & & & \\ & & \boxed{2} & & \\ & & & \boxed{2} & \\ & & & & \boxed{2} \end{pmatrix}$$

(3) Generalised eigenspaces:

$$\begin{aligned} \text{Ker}(A - \lambda_1 \mathbb{1})^2 &= \text{Ker} \left( \begin{pmatrix} 0 & 1 & -1 & 8 & -3 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 & 8 & -3 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right) \\ &= \text{Ker} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \dim(\text{Ker}(A - \lambda_1 \mathbb{1})^2) = \underline{5} \end{aligned}$$

2<sup>nd</sup> level

1<sup>st</sup> level



$\Rightarrow$

$$J = \begin{pmatrix} \boxed{2} & 1 & & & \\ & \boxed{2} & & & \\ & & \boxed{2} & & \\ & & & \boxed{2} & \\ & & & & \boxed{2} \end{pmatrix}$$