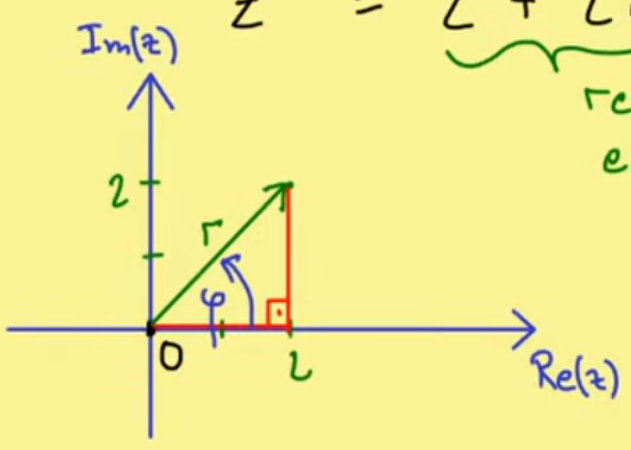
BECOME A MEMBER

ON STEADY

The Bright Side of
Mathematics

Solving equations with complex numbers

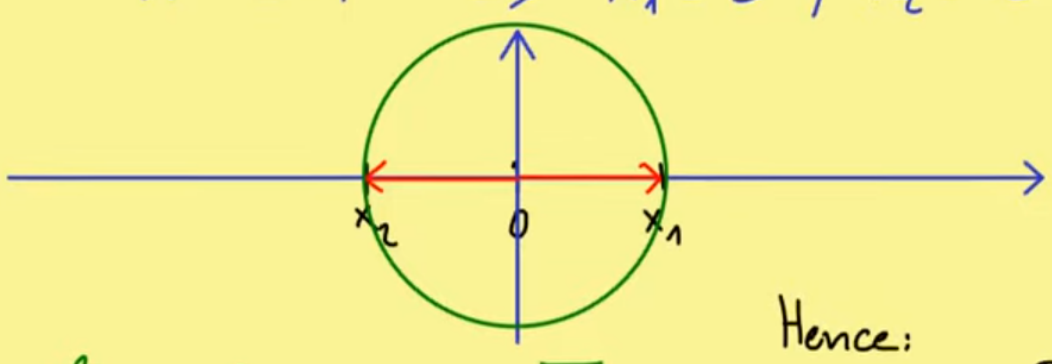
$z^3 = 2 + 2i$ rewrite in exponential form



$\varphi = \frac{\pi}{4}$

$r^2 = 2^2 + 2^2 = 8 \Rightarrow r = \sqrt{8}$

$x^2 = 4 \Rightarrow x_1 = 2, \; x_2 = -2$



Hence:

$2+2i = \sqrt{8} \cdot e^{i\frac{\pi}{4}}$

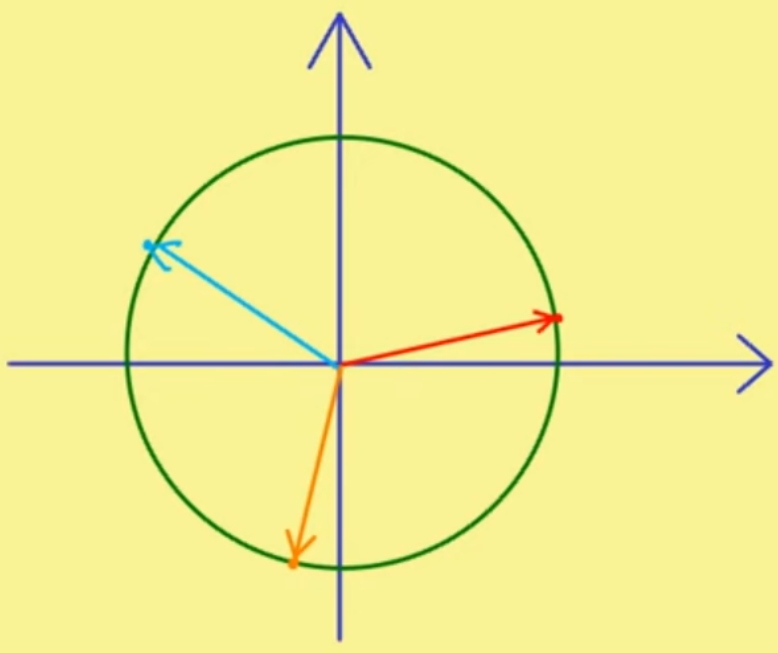
Solve $z^3 = \sqrt{8} \cdot e^{i\frac{\pi}{4}}$

$\Rightarrow z^3 = \sqrt{8} \cdot e^{i\left(\frac{\pi}{4} + 2\pi \cdot k\right)}$, $k = 0, 1, 2, \ldots$

$\Rightarrow z_k = (\sqrt{8})^{1/3} \, e^{i\left(\frac{\pi}{4} + 2\pi k\right) \cdot \frac{1}{3}}$

$\Rightarrow z_k = 8^{1/6} \, e^{i\left(\frac{\pi}{12} + \frac{2}{3}\pi \cdot k\right)}$

$k = 0, 1, 2$



$\Rightarrow z_0 = \sqrt{2} \, e^{i\frac{\pi}{12}}$

$z_1 = \sqrt{2} \, e^{i\frac{9}{12}\pi}$

$z_2 = \sqrt{2} \, e^{i\frac{17}{12}\pi}$

$z^4 = 3 + 2i$