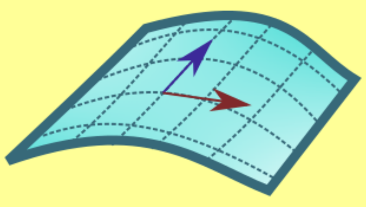
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics



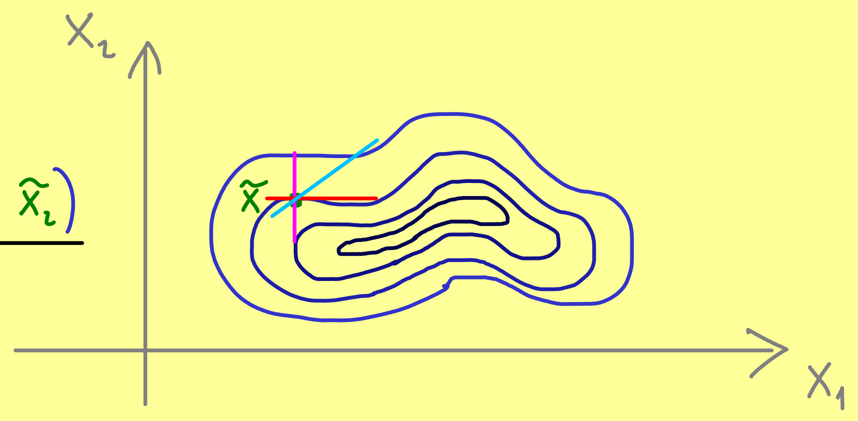
Multivariable Calculus – Part 10

$f: \mathbb{R}^n \to \mathbb{R}$

Different derivatives:
- partial derivatives $\frac{\partial f}{\partial x_i}(\tilde{x})$
- directional derivatives
- total derivative $J_f(\tilde{x})$, $\operatorname{grad} f(\tilde{x})$

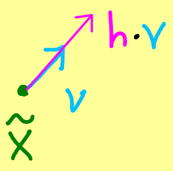
Visualisation for $f: \mathbb{R}^2 \to \mathbb{R}$

$$\frac{\partial f}{\partial x_2}(\tilde{x}) = \lim_{h \to 0} \frac{f(\tilde{x}_1, \tilde{x}_2 + h) - f(\tilde{x}_1, \tilde{x}_2)}{h}$$



Definition: For $f: \mathbb{R}^n \to \mathbb{R}$ and $\tilde{x} \in \mathbb{R}^n$, $v \in \mathbb{R}^n$ (unit vector), the limit

$$\lim_{h \to 0} \frac{f(\tilde{x} + h \cdot v) - f(\tilde{x})}{h}$$



is called the directional derivative of f along v at $\tilde{x}$.

Notations: $(\partial_v f)(\tilde{x})$, $(D_v f)(\tilde{x})$, $(\nabla_v f)(\tilde{x})$, $(v \cdot \nabla f)(\tilde{x})$

Proposition: $f: \mathbb{R}^n \to \mathbb{R}$ totally differentiable at $\tilde{x} \in \mathbb{R}^n$, $v \in \mathbb{R}^n$ (unit vector).

$$\lim_{h \to 0} \frac{f(\tilde{x} + h \cdot v) - f(\tilde{x})}{h} = \frac{d}{dt} f(\tilde{x} + t \cdot v)\Big|_{t=0} = \frac{d}{dt}(f \circ \gamma)(t)\Big|_{t=0}$$

(function: $t \mapsto f(\tilde{x} + t \cdot v)$) (curve: $\gamma(t) = \tilde{x} + t \cdot v$)

$$\overset{\text{chain rule}}{=} J_f(\gamma(t))\, J_\gamma(t)\Big|_{t=0} = J_f(\gamma(0))\, J_\gamma(0)$$

$$= J_f(\tilde{x})\, v = \langle \operatorname{grad} f(\tilde{x}), v \rangle$$