The Bright Side of Mathematics

Linear Algebra – Part 15

$A \in \mathbb{R}^{m \times n}$ ← collection of m row vectors

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = \begin{pmatrix} \text{—} \; \alpha_1^T \; \text{—} \\ \text{—} \; \alpha_2^T \; \text{—} \\ \vdots \\ \text{—} \; \alpha_m^T \; \text{—} \end{pmatrix}$$

$$\alpha_i^T := \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix}$$

T stands for "transpose"

flat matrix $\mathbb{R}^{1 \times n}$

$$u^T = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}^T = \begin{pmatrix} u_1 & u_2 & \cdots & u_n \end{pmatrix}$$

transpose of column vector = row vector

$u^T x$ for $x \in \mathbb{R}^n$ is defined.

Example:

$$\begin{pmatrix} 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} = 1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 = \left\langle \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \right\rangle$$

standard inner product

Remember: For $u, v \in \mathbb{R}^n$: $\quad u^T v = \langle u, v \rangle$

Row picture of the matrix-vector multiplication:

$$Ax = \begin{pmatrix} \text{—} \; \alpha_1^T \; \text{—} \\ \text{—} \; \alpha_2^T \; \text{—} \\ \vdots \\ \text{—} \; \alpha_m^T \; \text{—} \end{pmatrix} \underbrace{\begin{pmatrix} | \\ x \\ | \end{pmatrix}}_{\in \mathbb{R}^n} = \begin{pmatrix} \alpha_1^T x \\ \alpha_2^T x \\ \vdots \\ \alpha_m^T x \end{pmatrix} \in \mathbb{R}^m$$

Example:

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \cdot 3 + 1 \cdot 1 + 2 \cdot 0 \\ 3 \cdot 3 + 2 \cdot 1 + 1 \cdot 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$$