The Bright Side of Mathematics

Linear Algebra – Part 12

Example: Xavier is two years older than Yasmin. Together they are 40 years old.

How old is Xavier?
How old is Yasmin?

$$x = y + 2$$
$$x + y = 40$$

← two unknowns and two equations

Another Example:

$$\left.\begin{aligned} 2x_1 - 3x_2 + 4x_3 &= -7 \\ -3x_1 + x_2 - x_3 &= 0 \\ 20x_1 + 10x_2 &= 80 \\ 10x_2 + 25x_3 &= 90 \end{aligned}\right\}$$

4 equations and 3 unknowns x_1, x_2, x_3

Linear equation: $\text{constant} \cdot x_1 + \text{constant} \cdot x_2 + \cdots + \text{constant} \cdot x_n = \text{constant}$

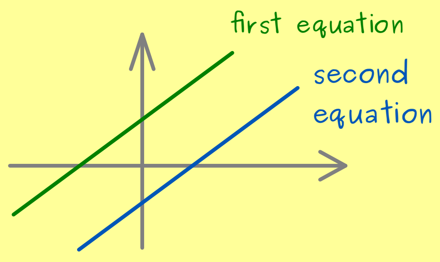
Definition: System of linear equations (LES) with m equations and n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad\quad \vdots \qquad \vdots \qquad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

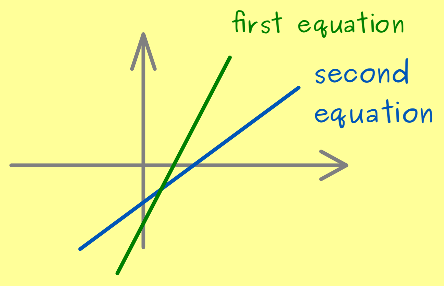
A solution of the LES: choice of values for $x_1, \dots, x_n$ such that all m equations are satisfied.

Note:

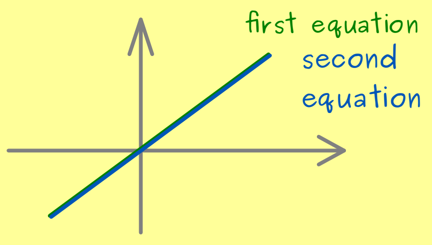
- it's possible that there is no solution $m = 2,\ n = 2$



- it's possible that there is a unique solution $m = 2,\ n = 2$



- it's possible that there are infinitely many solutions



Short notation: Instead of

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad\quad \vdots \qquad \vdots \qquad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

we write $Ax = b$ with $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$, $b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$

and $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

Example:

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 &= -7 \\ -3x_1 + x_2 - x_3 &= 0 \\ 20x_1 + 10x_2 &= 80 \\ 10x_2 + 25x_3 &= 90 \end{aligned}$$

can be written as

$$\begin{pmatrix} 2 & -3 & 4 \\ -3 & 1 & -1 \\ 20 & 10 & 0 \\ 0 & 10 & 25 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \\ 80 \\ 90 \end{pmatrix}$$

matrix-vector product

"matrix times vector = vector"