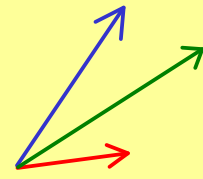




The Bright Side of Mathematics

Functional analysis - part 6



Definition: $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$. Let X be a \mathbb{F} -vector space.

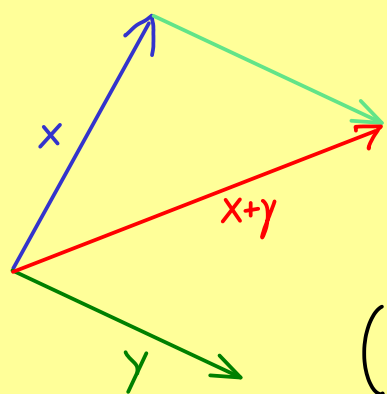
A map $\|\cdot\|: X \rightarrow [0, \infty)$ is called norm if

(a) $\|x\| = 0 \iff x = 0$ (positive definite)

(b) $\|\lambda \cdot x\| = |\lambda| \|x\|$ for all $\lambda \in \mathbb{F}$, $x \in X$ (absolutely homogeneous)
absolute value in \mathbb{R} or \mathbb{C}

$\xrightarrow{x} \xrightarrow{\lambda x}$

(c) $\|x+y\| \leq \|x\| + \|y\|$ for all $x, y \in X$ (triangle inequality)

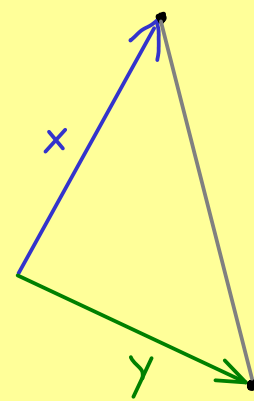


$(X, \|\cdot\|)$ is then called a normed space.

Important: If $\|\cdot\|$ is a norm for the \mathbb{F} -vector space X , then

$$d_{\|\cdot\|}(x, y) := \|x - y\| \text{ defines}$$

a metric for the set X .



If $(X, d_{\|\cdot\|})$ is a complete metric space,

then the normed space $(X, \|\cdot\|)$ is called a Banach space.

Banach space:

