

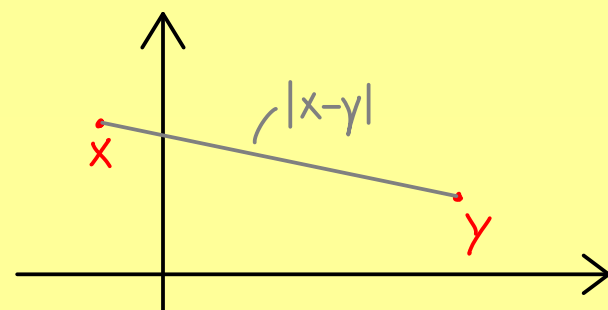


The Bright Side of Mathematics

Functional analysis - part 2

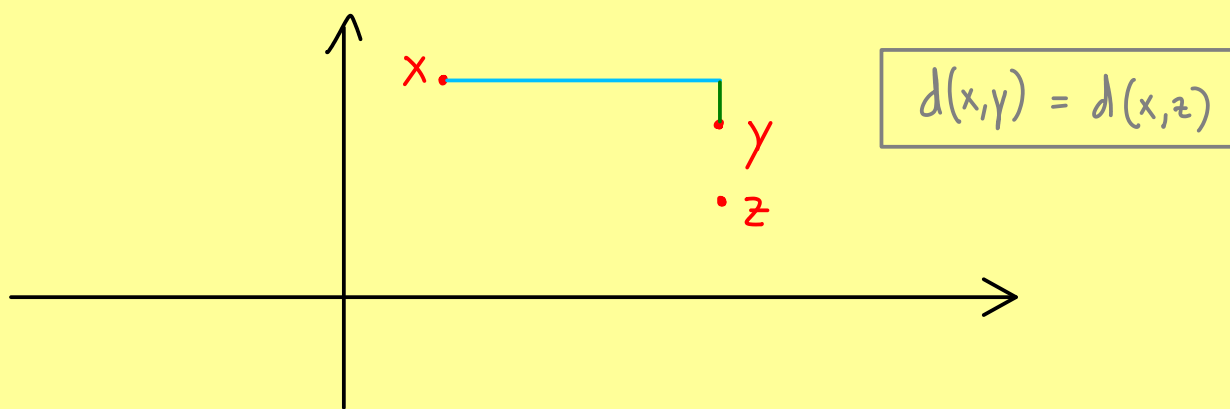
X set + $d: X \times X \rightarrow [0, \infty)$ metric = metric space (X, d)

Examples: (a) $X = \mathbb{C}$, $d(x, y) = |x - y|$



(b) $X = \mathbb{R}^n$, $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$ (Euclidean metric)

(c) $X = \mathbb{R}^n$, $d(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|, \dots, |x_n - y_n|\}$



(d) X any set ($\neq \emptyset$), $d(x, y) = \begin{cases} 0 & , x = y \\ 1 & , x \neq y \end{cases}$ discrete metric

d is a metric: (1) \checkmark , (2) \checkmark , (3) Δ -inequality: $x, y, z \in X$

First case: $x = y$: $d(x, y) = 0 \leq d(x, z) + d(z, y) \checkmark$

Second case: $x \neq y$: $d(x, y) = 1 = \begin{cases} d(x, z) \\ \text{or} \\ d(z, y) \end{cases} \leq d(x, z) + d(z, y) \checkmark$