



Distributions - Part 7

$$f \in \mathcal{L}'_{loc}(\mathbb{R}^n) \leftarrow \text{vector space of functions}$$

$$T_f \in \mathcal{D}'(\mathbb{R}^n) \leftarrow \text{vector space of distributions?}$$

Fact: $\mathcal{D}'(\mathbb{R}^n)$ is a real (or complex) vector space:

- addition: $+$ for $T, S \in \mathcal{D}'(\mathbb{R}^n)$, define $T + S \in \mathcal{D}'(\mathbb{R}^n)$

$$(T + S)(\varphi) = T(\varphi) + S(\varphi)$$

$$\underline{(T_f + T_g)(\varphi)} = T_f(\varphi) + T_g(\varphi)$$

$$= \int_{\mathbb{R}^n} f(x) \varphi(x) dx + \int_{\mathbb{R}^n} g(x) \varphi(x) dx$$

$$= \int_{\mathbb{R}^n} (f(x) + g(x)) \varphi(x) dx = \underline{T_{f+g}(\varphi)}$$

- scalar multiplication: • for $\lambda \in \mathbb{R}$ (or \mathbb{C}), $T \in \mathcal{D}'(\mathbb{R}^n)$
define $\lambda \cdot T$ by:

$$(\lambda \cdot T)(\varphi) = \lambda \cdot T(\varphi)$$

(we have all calculations rules in a vector space)

duality pairing: $\langle T, \varphi \rangle := T(\varphi)$

$$\langle \cdot, \cdot \rangle : \mathcal{D}'(\mathbb{R}^n) \times \mathcal{D}(\mathbb{R}^n) \longrightarrow \mathbb{R} \text{ (or } \mathbb{C}) \quad \text{bilinear map}$$