Aufgabe 11 Vektoranalysis - Kurven in $\mathbb{R}^n$

Betrachten Sie die folgende parametrisierte Kurve:

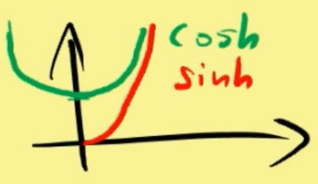
$$\gamma : [0, R] \to \mathbb{R}^3, \quad \gamma(t) = \begin{pmatrix} \cosh(t) \\ \sinh(t) \\ t \end{pmatrix}.$$

$R \in \mathbb{R}$.

(a) Skizzieren Sie die Kurve, d. h. das Bild der Abbildung.

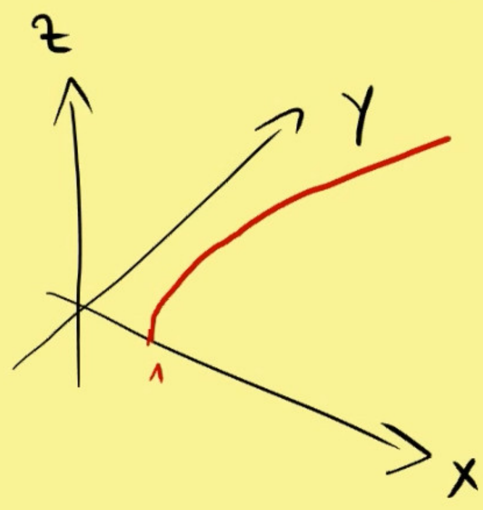
(b) Berechnen Sie die Bogenlänge der Kurve.

(a) $\gamma(t) = \begin{pmatrix} \cosh(t) \\ \sinh(t) \\ t \end{pmatrix}$



$\frac{1}{2}(e^x + e^{-x})$

$\frac{1}{2}(e^x - e^{-x})$



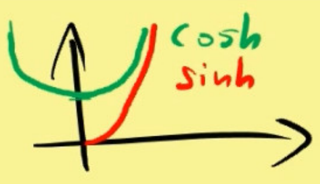
$\cosh^2(t) - \sinh^2(t)$

(b) $\dot{\gamma}(t) = \begin{pmatrix} \sinh(t) \\ \cosh(t) \\ 1 \end{pmatrix}, \quad \|\dot{\gamma}(t)\|^2 = \sinh^2(t) + \cosh^2(t) + 1$

$= 2\cosh^2(t) \quad \Rightarrow \|\dot{\gamma}(t)\| = \sqrt{2}\cosh(t)$

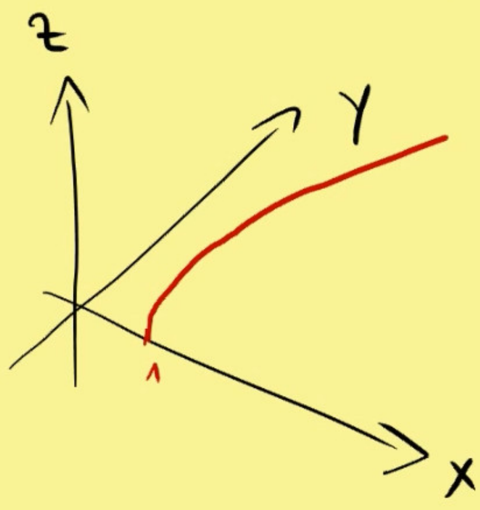
$$L[\gamma] = \int_0^R \|\dot{\gamma}(t)\|\, dt = \sqrt{2}\int_0^R \cosh(t)\, dt = \sqrt{2}\left[\sinh(t)\right]_0^R = \sqrt{2}\sinh(R)$$

(a) $\gamma(t) = \begin{pmatrix} \cosh(t) \\ \sinh(t) \\ t \end{pmatrix}$



$\frac{1}{2}(e^x + e^{-x})$

$\frac{1}{2}(e^x - e^{-x})$



$\cosh^2(t) - \sinh^2(t)$

(b) $\dot{\gamma}(t) = \begin{pmatrix} \sinh(t) \\ \cosh(t) \\ 1 \end{pmatrix}, \quad \|\dot{\gamma}(t)\|^2 = \sinh^2(t) + \cosh^2(t) + 1$

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