Unbounded Operators – Part 4

Closed operator: $T: X \supseteq \mathcal{D}(T) \longrightarrow Y$ closed

$\Longleftrightarrow$ $G_T := \{(x,y) \in X \times Y \mid x \in \mathcal{D}(T),\ Tx = y\}$ closed

Closable operator: $T: X \supseteq \mathcal{D}(T) \longrightarrow Y$ closable

$:\Longleftrightarrow$ $\overline{G_T}$ is the graph of an operator $\overline{T}$ ← closure of T

Proposition: $T: X \supseteq \mathcal{D}(T) \longrightarrow Y$ closable

$\Longleftrightarrow$ $\overline{G_T}$ is a graph (not possible $(0,0), (0,y) \in \overline{G_T}$ for $y \neq 0$)

$\Longleftrightarrow$ If $(0,y) \in \overline{G_T}$, then $y = 0$. $\boxed{G_T := \{(x,y) \in X \times Y \mid x \in \mathcal{D}(T),\ Tx = y\}}$

$\Longleftrightarrow$ For each $(x_n) \subseteq \mathcal{D}(T)$ with $x_n \to 0$ and $Tx_n \longrightarrow y$,
we have $y = 0$.

Define $\overline{T}$ for a closable operator $T: X \supseteq \mathcal{D}(T) \longrightarrow Y$:

$$\mathcal{D}(\overline{T}) := \{x \in X \mid \exists (x_n) \subseteq \mathcal{D}(T): x_n \to x \text{ and } Tx_n \text{ convergent}\}$$

$\overline{T}x := \lim_{n \to \infty} Tx_n$ operator! (closure of T)

$\Longrightarrow$ $T \subseteq \overline{T}$