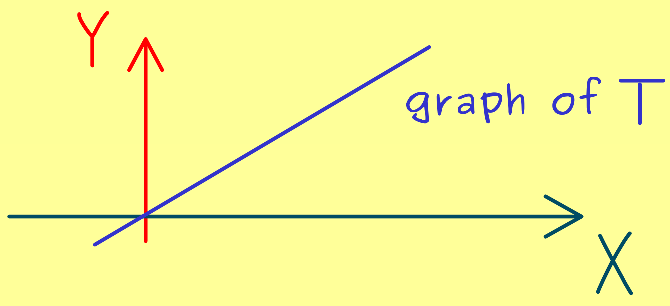
Unbounded Operators – Part 3

Recall: operator $T: X \supseteq D(T) \to Y$ (linear map between normed spaces)

$\leftrightsquigarrow$ subset in $X \times Y$



graph of T: $G_T := \{ (x,y) \in X \times Y \mid x \in D(T),\ Tx = y \}$

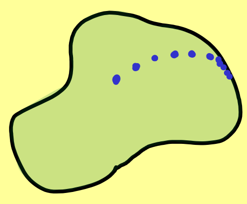
$X \times Y$ normed space with $\|(x,y)\|_{X\times Y} := \|x\|_X + \|y\|_Y$

Definition: An operator $T: X \supseteq D(T) \to Y$ is called a closed operator if the graph G_T is closed (in the normed space $X \times Y$).

Note: T closed $\iff$ $\boxed{\begin{array}{l} \text{for each sequence } (x_n) \subseteq D(T) \text{ with} \\ x_n \xrightarrow{n\to\infty} x \in X,\ Tx_n \to y \in Y, \\ \text{we have: } x \in D(T) \text{ and } Tx = y \end{array}}$

Proof: G_T closed $\iff$ for each sequence $(x_n, Tx_n) \subseteq G_T$ that is convergent in $X \times Y$ with limit $(x,y) \in X \times Y$,

we have: $\underbrace{(x,y) \in G_T}_{x \in D(T) \text{ and } Tx = y}$.



Remember: $T: X \to Y$ with $D(T) = X$ bounded $\Rightarrow$ closed operator