Tensor Analysis - Part 1

Linear Algebra → Tensor Analysis

Abstract Linear Algebra → Tensor Analysis

Multivariable Calculus → Tensor Analysis

Tensor Analysis ↔ Manifolds

- multilinear maps
- tensor product
- tensors (contravariant/covariant)
- tensor fields

Roughly explained concept:

	concrete (w.r.t. a basis)	abstract
	$\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^n \end{pmatrix} \in \mathbb{R}^n$	vector $v \in V$
matrix	$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & \cdots & & a_{mn} \end{pmatrix}$	linear map $\ell : V \longrightarrow W$
components	$\varphi^{\nu}_{\mu_1 \mu_2 \cdots \mu_n}$	multilinear map $\varphi : \underbrace{V \times V \times \cdots \times V}_{n \text{ times}} \longrightarrow W$
components	$\varphi^{\nu_1 \nu_2 \cdots \nu_m}_{\mu_1 \mu_2 \cdots \mu_n}$	multilinear map $\varphi : V^* \times \cdots \times V^* \times V \times \cdots \times V \to \mathbb{R}$ (dual space)
	tensors	tensors

Example: $\varepsilon_{\mu_1 \mu_2 \mu_3}$ Levi-Civita tensor (on $\{1,2,3\}$) :

$$\varepsilon_{\mu_1 \mu_2 \mu_3} := \begin{cases} 1 \, , & (\mu_1, \mu_2, \mu_3) \text{ even permutation} \\ -1 \, , & (\mu_1, \mu_2, \mu_3) \text{ odd permutation} \\ 0 \, , & \text{else} \end{cases}$$

for example: $(2,1,3)$ odd permuation

Visualization: