Tensor Analysis - Part 1

Linear Algebra → Tensor Analysis

Abstract Linear Algebra → Tensor Analysis

Multivariable Calculus → Tensor Analysis

Tensor Analysis ↔ Manifolds

Tensor Analysis:

- multilinear maps
- tensor product
- tensors (contravariant/covariant)
- tensor fields

Roughly explained concept:

	concrete (w.r.t. a basis)	abstract
	$\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^n \end{pmatrix} \in \mathbb{R}^n$	vector $v \in V$
matrix	$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & & \vdots \\ a_{m1} & \cdots & & a_{mn} \end{pmatrix}$	linear map $\ell: V \longrightarrow W$
components	$\varphi^{\nu}_{\mu_1\mu_2\cdots\mu_n}$	multilinear map $\varphi: \underbrace{V\times V\times\cdots\times V}_{n\text{ times}} \longrightarrow W$
components	$\varphi^{\nu_1\nu_2\cdots\nu_m}_{\mu_1\mu_2\cdots\mu_n}$	multilinear map $\varphi: \underset{\text{dual space}}{V^*}\times\cdots\times V^*\times V\times\cdots\times V \longrightarrow \mathbb{R}$
	tensors	tensors

Example: $\varepsilon_{\mu_1\mu_2\mu_3}$ Levi-Civita tensor (on $\{1,2,3\}$):

$$\varepsilon_{\mu_1\mu_2\mu_3} := \begin{cases} 1, & (\mu_1,\mu_2,\mu_3) \text{ even permutation} \\ -1, & (\mu_1,\mu_2,\mu_3) \text{ odd permutation} \\ 0, & \text{else} \end{cases}$$

for example: $(2,1,3)$ odd permuation

Visualization: