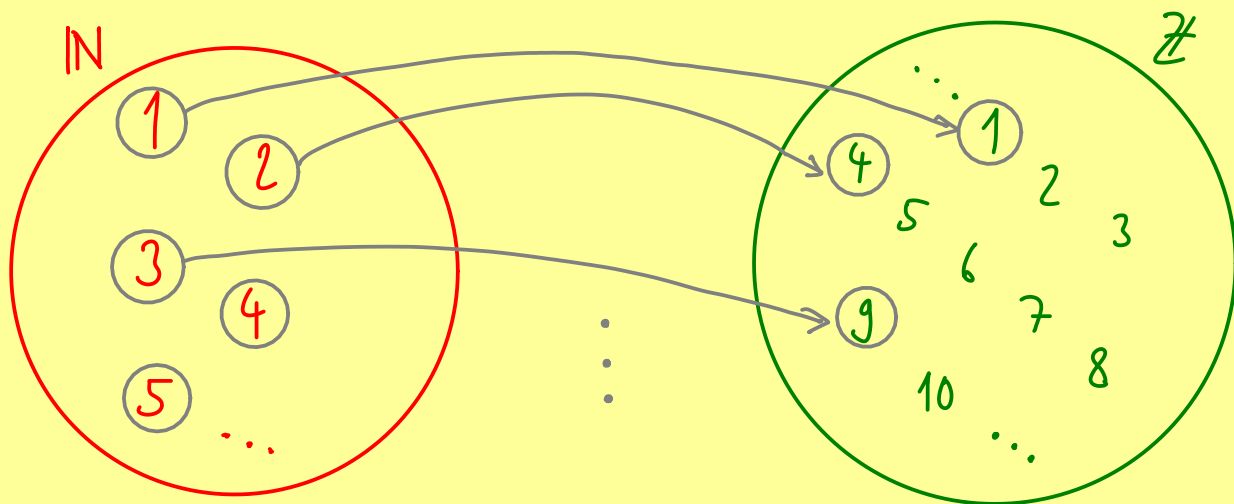The Bright Side of
Mathematics

Start Learning Sets – Part 5

Map: $f : A \longrightarrow B$

Example: $f : \mathbb{N} \longrightarrow \mathbb{Z}$

$x \longmapsto x^2$ — new notation for $f(x) = x^2$

Range: $Ran(f) := \left\{ y \in B \mid \exists x \in A : f(x) = y \right\}$

$=: \left\{ f(x) \mid x \in A \right\}$ (shorter notation)

Example: $f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$
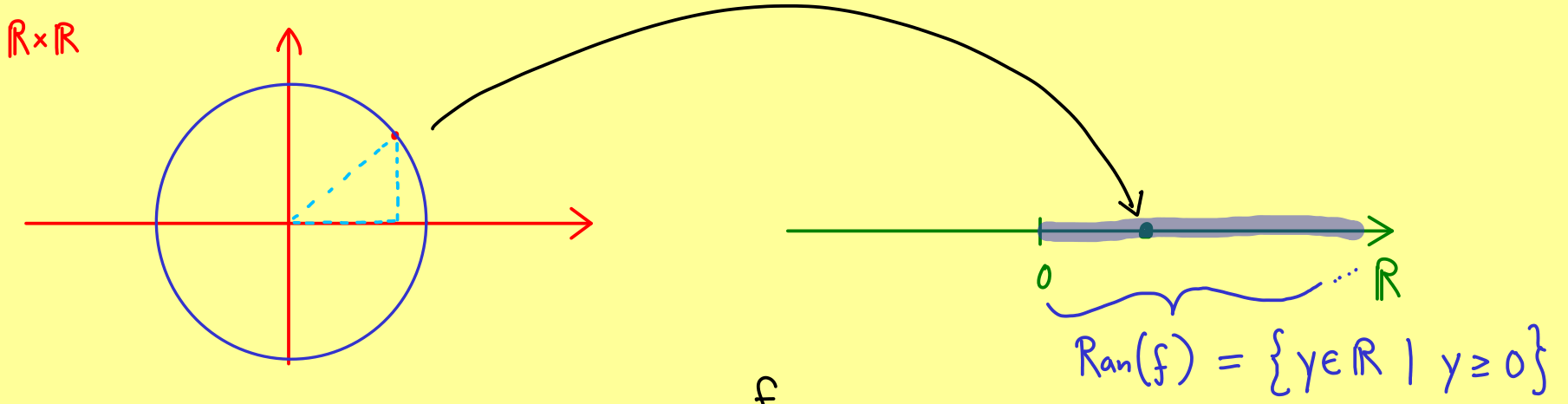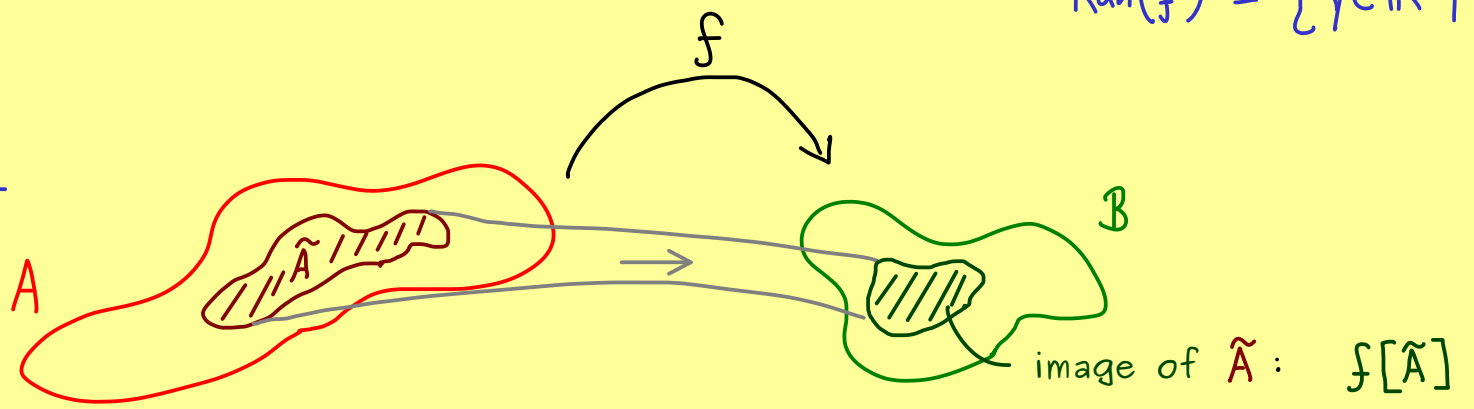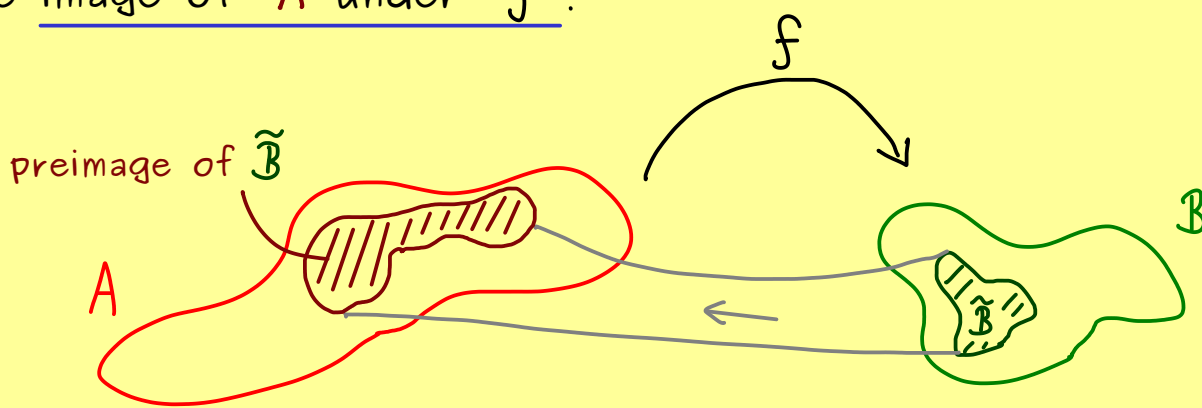
$(x_1, x_2) \longmapsto x_1^2 + x_2^2$

$\mathbb{R} \times \mathbb{R}$

$Ran(f) = \left\{ y \in \mathbb{R} \mid y \geq 0 \right\}$

Image and preimage:

image of $\tilde{A}$: $f[\tilde{A}]$

For a subset $\tilde{A} \subseteq A$,

$$f[\tilde{A}] := \left\{ y \in B \mid \exists x \in \tilde{A} : f(x) = y \right\} = \left\{ f(x) \mid x \in \tilde{A} \right\}$$

denotes the image of $\tilde{A}$ under f.

preimage of $\tilde{B}$

For $\tilde{B} \subseteq B$,

$$f^{-1}[\tilde{B}] := \left\{ x \in A \mid f(x) \in \tilde{B} \right\}$$

denotes the preimage of $\tilde{B}$ under f.

Example: $f : \mathbb{N} \longrightarrow \mathbb{Z}$

$x \longmapsto \begin{cases} 0 & \text{if } x \text{ even} \\ x & \text{if } x \text{ odd} \end{cases}$

$f[\{2,3,4\}] = \{0,3\}$

$f^{-1}[\{0\}] = \{2, 4, 6, 8, 10, \ldots\}$