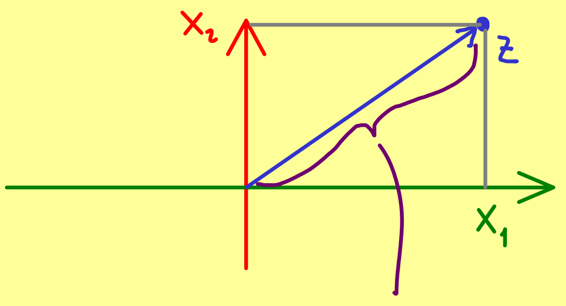
Start Learning Complex Numbers – Part 3

$z = x_1 + i \cdot x_2 \in \mathbb{C}$

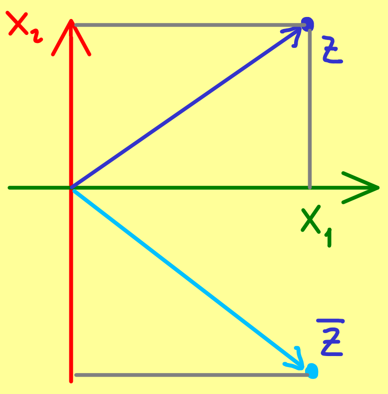
real part of z: $\mathrm{Re}(z)$ — imaginary part of z: $\mathrm{Im}(z)$



length, absolute value, modulus

$$|z| := \sqrt{\mathrm{Re}(z)^2 + \mathrm{Im}(z)^2} \in \mathbb{R}$$

Reflection:
complex conjugate



$$z = x_1 + i \cdot x_2$$

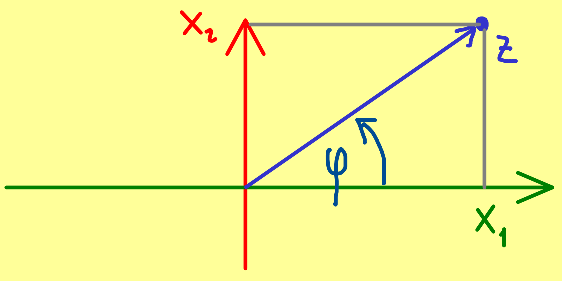
$$\bar{z} = x_1 + i \cdot (-x_2) = x_1 - i \cdot x_2$$

Calculate: $z \cdot \bar{z} = (x_1 + i \cdot x_2) \cdot (x_1 - i \cdot x_2)$

$$= x_1^2 + x_1 \cdot (-i \cdot x_2) + i \cdot x_2 x_1 - i^2 \cdot x_2^2$$

$$= x_1^2 + x_2^2 = |z|^2$$

Polar coordinates:



length: $|z|$

angle: $\varphi \in [0, 2\pi)$

argument of z:

$$\varphi = \begin{cases} \arctan\left(\frac{x_2}{x_1}\right) & , x_1 > 0 , \; x_2 \geq 0 \\ \frac{\pi}{2} & , x_1 = 0 , \; x_2 > 0 \\ \arctan\left(\frac{x_2}{x_1}\right) + \pi & , x_1 < 0 \\ \frac{3\pi}{2} & , x_1 = 0 , \; x_2 < 0 \\ \arctan\left(\frac{x_2}{x_1}\right) + 2\pi & , x_1 > 0 , \; x_2 < 0 \end{cases}$$

$$z = x_1 + i \cdot x_2 = |z| \cdot \big(\cos(\varphi) + i \cdot \sin(\varphi)\big)$$

Example: $z = 3 + i \cdot 3$, $\bar{z} = 3 - i \cdot 3$, $z \cdot \bar{z} = 9 + 9 = 18$

$\Rightarrow |z| = \sqrt{18} = 3 \cdot \sqrt{2}$, $\varphi = \arctan\left(\frac{3}{3}\right) = \frac{\pi}{4}$

$\Rightarrow z = 3 \cdot \sqrt{2} \cdot \left(\cos\left(\frac{\pi}{4}\right) + i \cdot \sin\left(\frac{\pi}{4}\right)\right) \overset{\text{later}}{=} 3 \cdot \sqrt{2} \cdot e^{i \frac{\pi}{4}}$