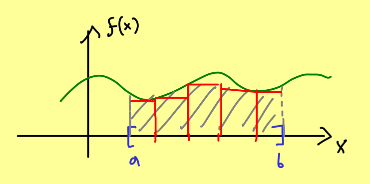
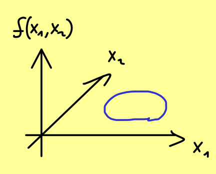
Riemann integral vs. Lebesgue integral

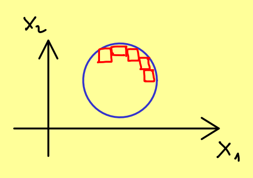
Riemann integral:

$f: \mathbb{R}^2 \to \mathbb{R}$





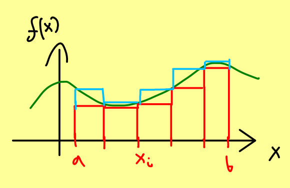
Problems of Riemann integral:



- difficult to expand to higher dimensions
- dependence on continuity
- limit processes

$$\lim_{n\to\infty} \int_a^b f_n(x)\,dx \overset{?}{=} \int_a^b \lim_{n\to\infty} f_n(x)\,dx$$

Riemann integral:



$f: \mathbb{R} \to \mathbb{R}$

$U(P) = \sum \sup f(x) \cdot \Delta x_i$

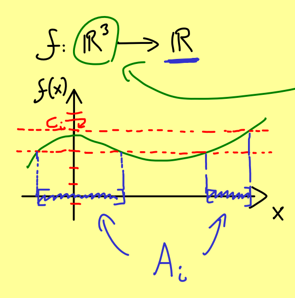
$L(P) = \sum \inf f(x) \cdot \Delta x_i$

Lebesgue integral:

$f: \Omega \to \mathbb{R}$

$f: \mathbb{R}^3 \to \mathbb{R}$

measure (volumes?) μ



$$\sum_i c_i \cdot \mu(A_i) \rightsquigarrow \int f\,d\mu$$

Lebesgue integral