



The Bright Side of Mathematics

Real Analysis - Part 53

$$\mathcal{R}([a, b]) := \left\{ f: [a, b] \rightarrow \mathbb{R} \text{ bounded} \mid f \text{ Riemann-integrable} \right\}$$

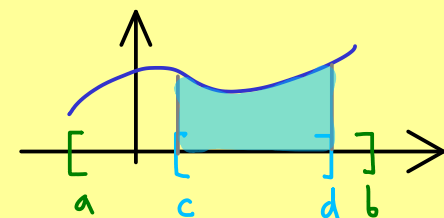
Property (1): map: $\mathcal{R}([a, b]) \rightarrow \mathbb{R}$

$$f \mapsto \int_a^b f(x) dx$$

is linear and monotonic

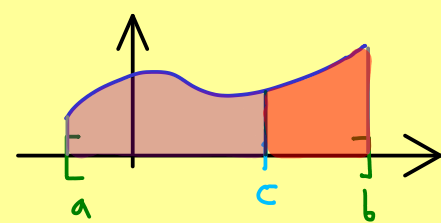
Definition: For $c, d \in [a, b]$ with $c < d$,

$$\int_c^d f(x) dx := \int_c^d f|_{[c, d]}(x) dx$$



Property (2): For $c \in [a, b]$, we have

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$



Definition: $\int_b^a f(x) dx := -\int_a^b f(x) dx$

Property (3): $f \in C([a, b]) \Rightarrow f \in \mathcal{R}([a, b])$

f monotonically increasing $\Rightarrow f \in \mathcal{R}([a, b])$