The Bright Side of
Mathematics

Real Analysis – Part 15

Series: "infinite sum", special sequence

$$a_1 + a_2 + a_3 + a_4 + \cdots = \sum_{k=1}^{\infty} a_k$$

Example: sequence $(a_k)_{k\in\mathbb{N}} = \left((-1)^k\right)_{k\in\mathbb{N}}$

$$\sum_{k=1}^{\infty} a_k = \underbrace{(-1+1)}_{=0} + \underbrace{((-1)+1)}_{=0} + \underbrace{((-1)+1)}_{=0} + \underbrace{((-1)+1)}_{=0} + (-1) + \cdots \overset{?}{=} 0$$

$$\sum_{k=1}^{\infty} a_k = -1 + \underbrace{(1+(-1))}_{=0} + \underbrace{(1+(-1))}_{=0} + \underbrace{(1+(-1))}_{=0} + \underbrace{(1+(-1))}_{=0} + \cdots \overset{?}{=} -1$$

Definition: Let $(a_k)_{k\in\mathbb{N}}$ be a sequence. The sequence $(S_n)_{n\in\mathbb{N}}$ given by

$$S_n := \sum_{k=1}^{n} a_k$$

is called a series.

If $(S_n)_{n\in\mathbb{N}}$ is convergent, we write:

$$\sum_{k=1}^{\infty} a_k := \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

Example from above: $\left(\sum_{k=1}^{n} (-1)^k\right)_{n\in\mathbb{N}} = (-1, 0, -1, 0, -1, 0, -1, \ldots)$

not convergent!

Another example: $\left(\sum_{k=1}^{n} (1)^k\right)_{n\in\mathbb{N}} = (1, 2, 3, 4, \ldots)$ divergent to ∞