



# The Bright Side of Mathematics

## Real Analysis - Part 1

Calculus, analysis, infinitesimal calculus, ...

Goal: Understanding differential and integral calculations

$$\sim \frac{df}{dx}, \int_a^b f dx$$

Topics: Limits, continuity, derivatives, integrals.

Foundation: Real numbers:  $\mathbb{R} \rightsquigarrow$  Start Learning Mathematics  
 $\hookrightarrow$  Start Learning Reals

Axioms of the reals: A non-empty set  $\mathbb{R}$  together with operations  $+$ ,  $\cdot$  and ordering  $\leq$  is called the real numbers if it satisfies:

(A)  $(\mathbb{R}, +, 0)$  is an abelian group

(M)  $(\mathbb{R} \setminus \{0\}, \cdot, 1)$  is an abelian group ( $1 \neq 0$ )

(D) distributive law  $x \cdot (y + z) = x \cdot y + x \cdot z$

(O)  $\leq$  is a total order, compatible with  $+$  and  $\cdot$ , Archimedean property

(C) Every Cauchy sequence is a convergent sequence.  $|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Absolute value:

$$|x \cdot y| = |x| \cdot |y|$$

$$|x + y| \leq |x| + |y|$$

