

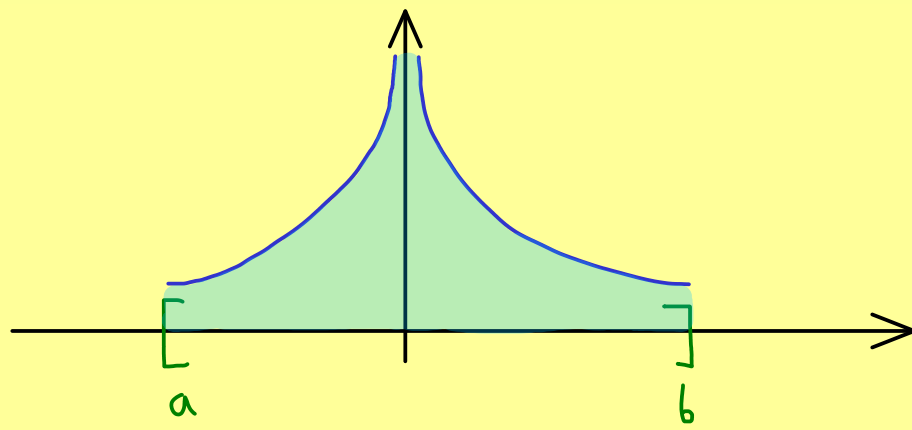


The Bright Side of Mathematics

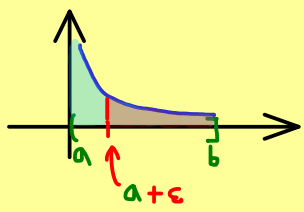
Real Analysis - Part 63

We know: $f: [a, b] \rightarrow \mathbb{R}$ Riemann-integrable
 $\Rightarrow f$ is bounded

What about this?



Definition: Let $f: (a, b] \rightarrow \mathbb{R}$ be a function with the property that



$f|_{[a+\epsilon, b]} \in \mathcal{R}([a+\epsilon, b])$ for all $\epsilon > 0$.

If $\lim_{\epsilon \searrow 0} \int_{a+\epsilon}^b f(x) dx$ exists, we write $\int_a^b f(x) dx$ for this limit and we say the integral converges.

Example:

$$\int_0^1 \log(x) dx ?$$

integration
by parts

$$\begin{array}{l} f'(x) = 1 \\ g(x) = \log(x) \\ f(x) = x \\ g'(x) = \frac{1}{x} \end{array}$$

$$\int_{\epsilon}^1 \log(x) dx = \int_{\epsilon}^1 1 \cdot \log(x) dx = x \cdot \log(x) \Big|_{\epsilon}^1 - \int_{\epsilon}^1 x \cdot \frac{1}{x} dx = x \cdot (\log(x) - 1) \Big|_{\epsilon}^1$$

$$\lim_{\epsilon \searrow 0} \int_{\epsilon}^1 \log(x) dx = 1 \cdot (\log(1) - 1) - \lim_{\epsilon \searrow 0} \epsilon \cdot (\log(\epsilon) - 1)$$

$$= -1 - \lim_{\epsilon \searrow 0} \epsilon \cdot \log(\epsilon) = -1 - \lim_{\epsilon \searrow 0} \frac{\log(\epsilon)}{\frac{1}{\epsilon}} = -1$$

