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ON STEADY

The Bright Side of
Mathematics

# Real Analysis – Part 58

## Integration by parts

$I \subseteq \mathbb{R}$ interval, $f, g : I \to \mathbb{R}$ continuously differentiable, $a, b \in I$

Then: $$\int_a^b f'(x)\cdot g(x)\,dx = f(x)\cdot g(x)\Big|_{x=a}^{x=b} - \int_a^b f(x)\cdot g'(x)\,dx$$

Example: $$\int_a^b \overset{g(x)}{\overbrace{x}}\cdot\underbrace{\exp(x)}_{f'(x)}\,dx = x\cdot\exp(x)\Big|_{x=a}^{x=b} - \int_a^b \exp(x)\cdot 1\,dx$$

$$= x\cdot\exp(x)\Big|_{x=a}^{x=b} - \exp(x)\Big|_{x=a}^{x=b}$$

$$= \big(x\cdot\exp(x) - \exp(x)\big)\Big|_{x=a}^{x=b}$$

$f'(x) = \exp(x)$
$g(x) = x$
$f(x) = \exp(x)$
$g'(x) = 1$

Proof: product rule: $(f\cdot g)'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$$f(x)\cdot g(x)\Big|_{x=a}^{x=b} \overset{\substack{\text{fundamental}\\ \text{theorem}\\ \text{of calculus}}}{=} \int_a^b (f\cdot g)'(x)\,dx = \int_a^b f'(x)\cdot g(x)\,dx + \int_a^b f(x)\cdot g'(x)\,dx \quad \square$$