The Bright Side of Mathematics

Real Analysis – Part 55

Proposition: I interval, $f: I \longrightarrow \mathbb{R}$ continuous,

$F: I \longrightarrow \mathbb{R}$ antiderivative of f.

Then: $G: I \longrightarrow \mathbb{R}$ antiderivative of f

$\Leftrightarrow$ $F - G$ is constant

Proof: ($\Rightarrow$) $F, G: I \longrightarrow \mathbb{R}$ two antiderivatives of f

$$(F-G)' = F' - G' = f - f = 0 \overset{\text{mean value theorem}}{\Longrightarrow} F - G \text{ is constant}$$

($\Leftarrow$) $F - G$ is constant $\Rightarrow$ $F(x) - G(x) = c$ for a number $c \in \mathbb{R}$

$\Rightarrow G = F - c \Rightarrow G' = F' = f \Rightarrow G$ antiderivative of f

Theorem: I interval, $f: I \longrightarrow \mathbb{R}$ continuous, $F: I \longrightarrow \mathbb{R}$ antiderivative of f.

second fundamental theorem of calculus

Then: $$\int_a^b f(t)\,dt = F(b) - F(a) =: F(x)\Big|_a^b$$

Example: $$\int_0^1 \cos(x)\,dx = \sin(x)\Big|_0^1 = \sin(1)$$