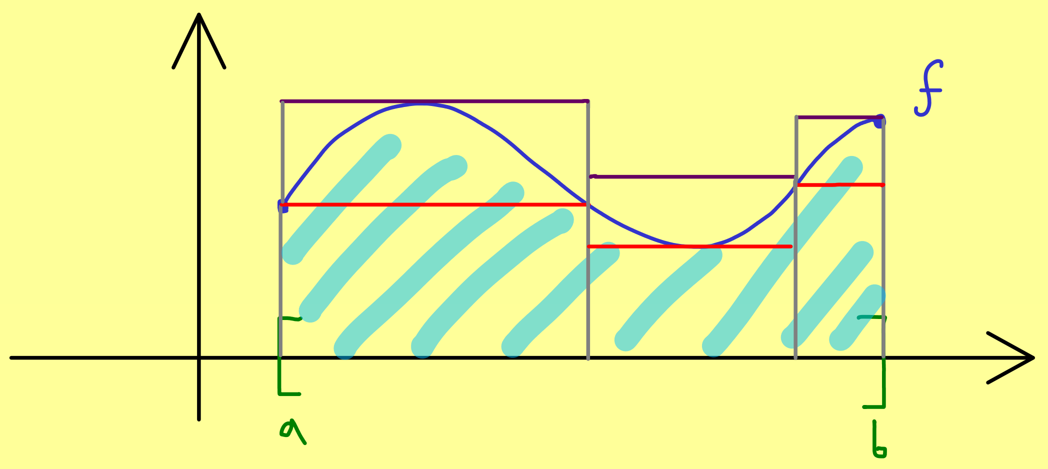
The Bright Side of Mathematics

Real Analysis – Part 51



$f: [a,b] \longrightarrow \mathbb{R}$
bounded

Use step functions $\phi \in \mathcal{S}([a,b])$:

$$\sup \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in \mathcal{S}([a,b]),\ \phi \leq f \right\}$$

$$\inf \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in \mathcal{S}([a,b]),\ \phi \geq f \right\}$$

Definition: A bounded function $f: [a,b] \longrightarrow \mathbb{R}$ is called Riemann-integrable if

$$\sup \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in \mathcal{S}([a,b]),\ \phi \leq f \right\} = \inf \left\{ \int_a^b \phi(x)\, dx \;\middle|\; \phi \in \mathcal{S}([a,b]),\ \phi \geq f \right\}$$

In this case: $\int_a^b f(x)\, dx$ is called the (Riemann) integral of f