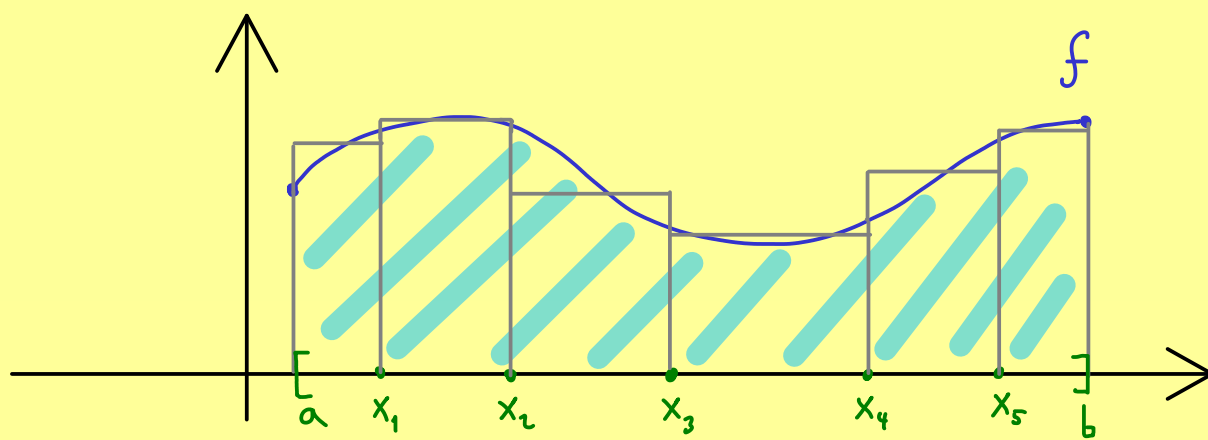




The Bright Side of Mathematics

Real Analysis - Part 48



(orientated)
area between
graph and x-axis

$$\sum_{j=1}^n f(\xi_j) \cdot (x_j - x_{j-1})$$

$\downarrow n \rightarrow \infty$

$$\int_a^b f(x) dx$$

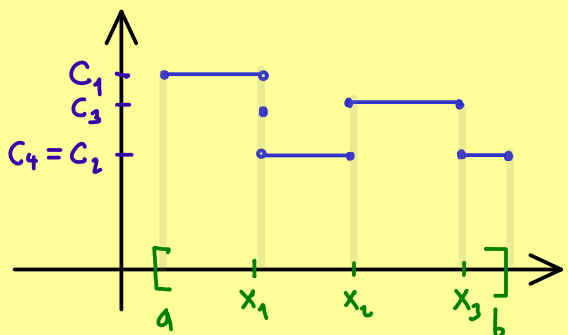
partition of x-axis \rightsquigarrow Riemann integral

(more modern: Lebesgue integral)

Definition: partition of $[a, b]$: a set $\{x_0, x_1, \dots, x_n\}$ with:

$$a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$$

Definition: $\phi: [a, b] \rightarrow \mathbb{R}$ is called a step function if it is piecewisely constant:



there is a partition of $[a, b]$, $\{x_0, x_1, \dots, x_n\}$,
and there are numbers $c_1, \dots, c_n \in \mathbb{R}$ such that

$$\phi|_{(x_{j-1}, x_j)} = c_j \quad \text{for all } j \in \{1, \dots, n\}$$

Can we define: $\int_a^b \phi(x) dx := \sum_{j=1}^n c_j \cdot (x_j - x_{j-1})$?