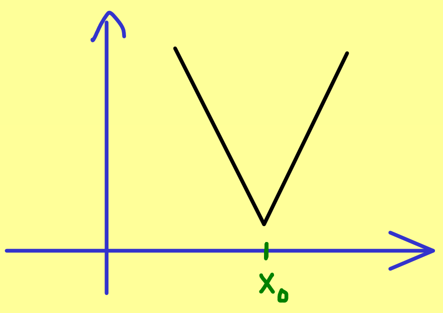
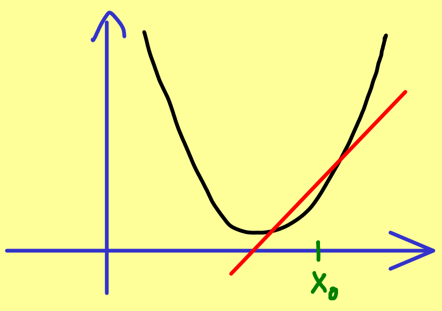
The Bright Side of Mathematics

Real Analysis – Part 34

Differentiability (linearisation) smoothness

$f: \mathbb{R} \to \mathbb{R}$





slope at point x_0 ?

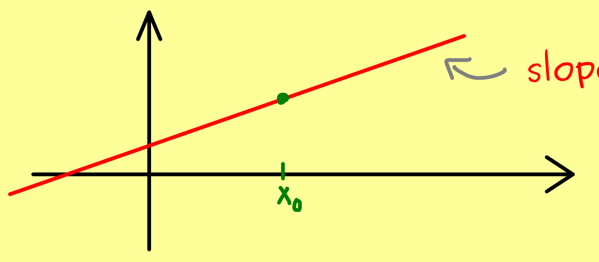
approximate f locally with a linear function?

(affine) linear function: (linear polynomial)

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = a_1 \cdot x + a_0 = m \cdot (x - x_0) + c$

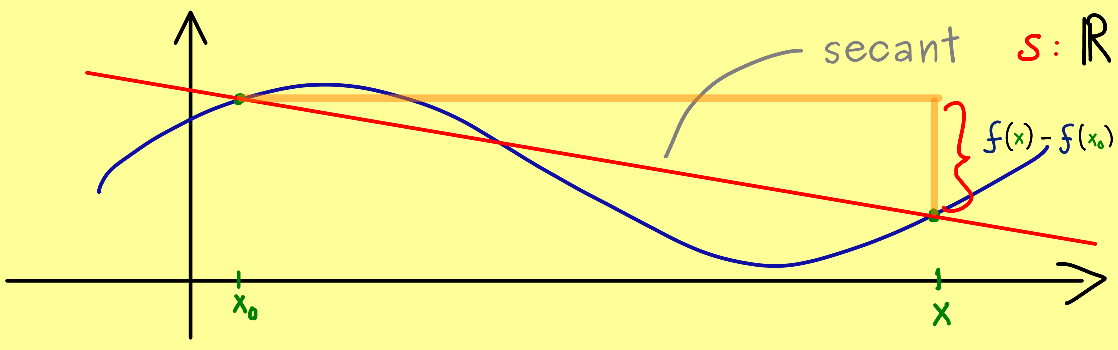
slope

constant $c = g(x_0)$



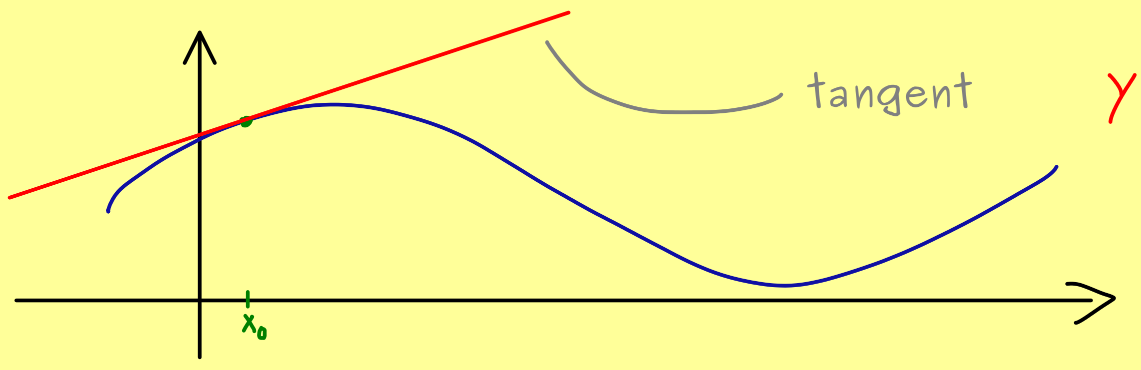
$\Rightarrow \quad m = \dfrac{g(x) - g(x_0)}{x - x_0}, \quad x \neq x_0$

Linear approximation: $f: \mathbb{R} \to \mathbb{R}$, $x_0 \in \mathbb{R}$



$s: \mathbb{R} \to \mathbb{R}$, $s(t) = m \cdot (t - x_0) + c$

$s(t) = \dfrac{f(x) - f(x_0)}{x - x_0}(t - x_0) + f(x_0)$



$y: \mathbb{R} \to \mathbb{R}$

$y(t) = \underbrace{\lim_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}}_{\text{we want it to exist}} (t - x_0) + f(x_0)$

slope at x_0: $\quad f'(x_0) := \lim_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0} =: \dfrac{df}{dx}(x_0)$ differential quotient/ derivative

Definition: $I \subseteq \mathbb{R}$ interval with more than one point

or $I \subseteq \mathbb{R}$ open set, $f: I \to \mathbb{R}$, $x_0 \in I$.

We call f differentiable at x_0 if there is a function $\Delta_{f,x_0}: I \to \mathbb{R}$

with $f(x) = f(x_0) + (x - x_0) \cdot \Delta_{f,x_0}(x)$ for all $x \in I$

and Δ_{f,x_0} is continuous at x_0.