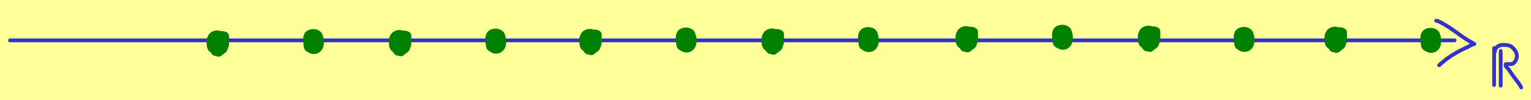
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Real Analysis – Part 11

Example: $(a_n)_{n\in\mathbb{N}}$ given by $a_n = n$



$\lim\limits_{n\to\infty} a_n = \infty$ $:\Longleftrightarrow$ divergent to ∞ $:\Longleftrightarrow$ $\forall C>0 \ \exists N\in\mathbb{N} \ \forall n\geq N : a_n > C$

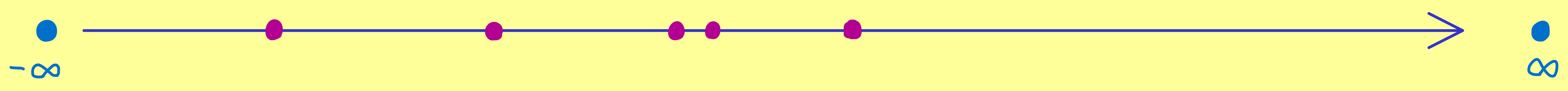
$\lim\limits_{n\to\infty} a_n = -\infty$ $:\Longleftrightarrow$ divergent to $-\infty$ $:\Longleftrightarrow$ $\forall C<0 \ \exists N\in\mathbb{N} \ \forall n\geq N : a_n < C$

$(a_n)_{n\in\mathbb{N}}$ has the improper accumulation value ∞ $:\Longleftrightarrow$ $(a_n)_{n\in\mathbb{N}}$ is not bounded from above

$(a_n)_{n\in\mathbb{N}}$ has the improper accumulation value $-\infty$ $:\Longleftrightarrow$ $(a_n)_{n\in\mathbb{N}}$ is not bounded from below

A given sequence $(a_n)_{n\in\mathbb{N}}$ could have many accumulation values:

and none, one or two improper accumulation values



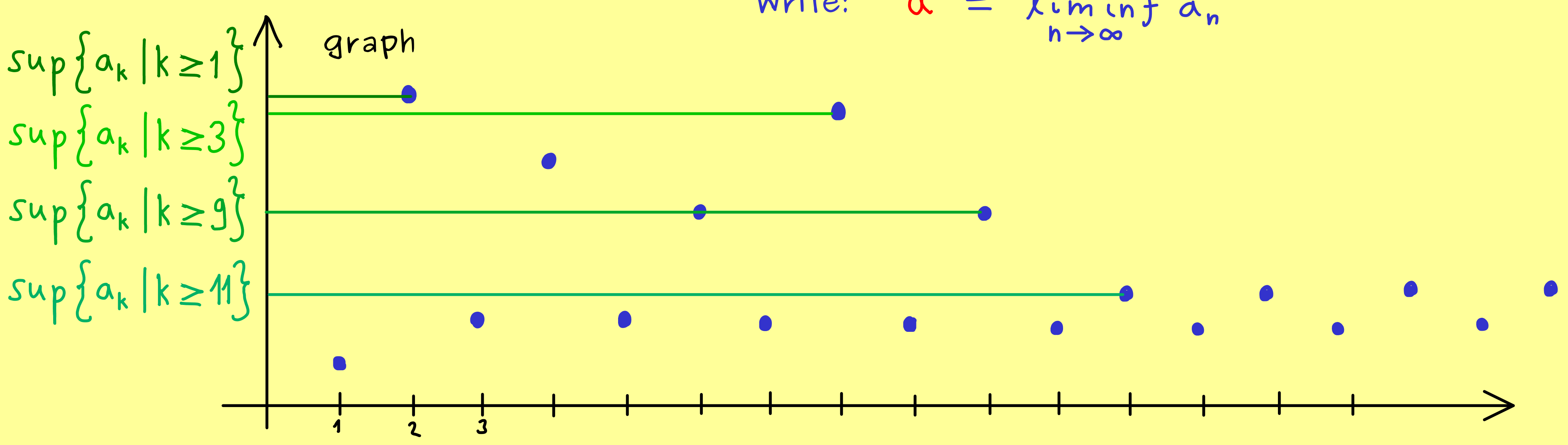
Definition: Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of real numbers. An element $a\in\mathbb{R}\cup\{-\infty,\infty\}$ is called:

- limit superior of $(a_n)_{n\in\mathbb{N}}$ if a is the largest (improper) accumulation value of $(a_n)_{n\in\mathbb{N}}$

  Write: $a = \limsup\limits_{n\to\infty} a_n$

- limit inferior of $(a_n)_{n\in\mathbb{N}}$ if a is the smallest (improper) accumulation value of $(a_n)_{n\in\mathbb{N}}$

  Write: $a = \liminf\limits_{n\to\infty} a_n$



Fact: $\limsup\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \sup\{a_k \mid k\geq n\}$

$\liminf\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \inf\{a_k \mid k\geq n\}$