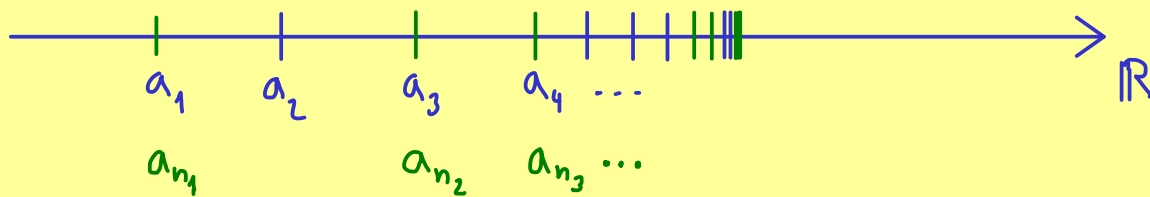


The Bright Side of Mathematics

Real Analysis - Part 9



Definition: Let $(n_k)_{k \in \mathbb{N}}$ be a sequence of natural numbers that is strictly monotonically increasing, then $(a_{n_k})_{k \in \mathbb{N}}$ is called a subsequence of $(a_n)_{n \in \mathbb{N}}$. $(\forall k \in \mathbb{N}: n_{k+1} > n_k)$

Example: $(a_n)_{n \in \mathbb{N}}$ given by $a_n = \frac{1}{n}$, $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \dots)$

$(a_{n_k})_{k \in \mathbb{N}} = (a_{2^k})_{k \in \mathbb{N}} = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots)$

Fact: $(a_n)_{n \in \mathbb{N}}$ convergent with $\lim_{n \rightarrow \infty} a_n = a$

\Rightarrow every subsequence $(a_{n_k})_{k \in \mathbb{N}}$ is convergent $\lim_{k \rightarrow \infty} a_{n_k} = a$

Example: $(a_n)_{n \in \mathbb{N}}$ given by $a_n = (-1)^n$

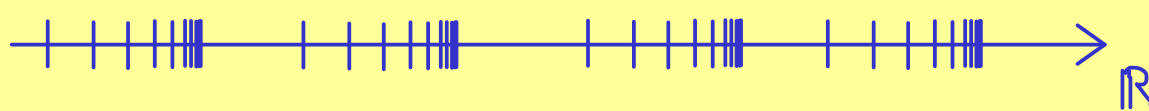
subsequence: $(a_{n_k})_{k \in \mathbb{N}} = (a_{2 \cdot k})_{k \in \mathbb{N}} = (1, 1, 1, 1, 1, \dots)$ limit 1

subsequence: $(a_{n_k})_{k \in \mathbb{N}} = (a_{2 \cdot k + 1})_{k \in \mathbb{N}} = (-1, -1, -1, \dots)$ limit -1 accumulation values

Definition: $a \in \mathbb{R}$ is called an accumulation value of $(a_n)_{n \in \mathbb{N}}$

if there is a subsequence $(a_{n_k})_{k \in \mathbb{N}}$ with $\lim_{k \rightarrow \infty} a_{n_k} = a$

(cluster point, accumulation point, limit point, partial limit, ...)



Show:

$a \in \mathbb{R}$ is an accumulation value of $(a_n)_{n \in \mathbb{N}}$ $\Leftrightarrow (a - \epsilon, a + \epsilon)$

$\Leftrightarrow \forall \epsilon > 0$: The ϵ -neighbourhood of a contains infinitely many sequence members of $(a_n)_{n \in \mathbb{N}}$