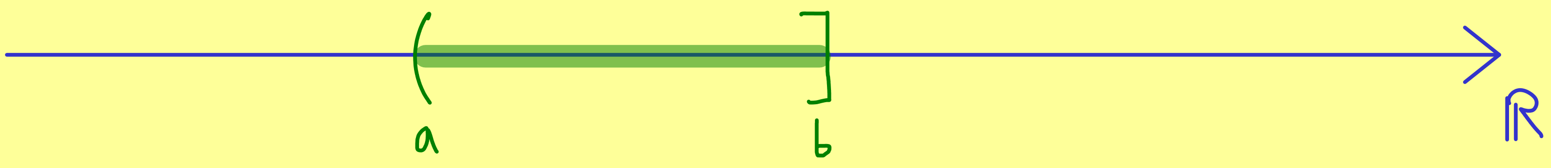
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Real Analysis – Part 6

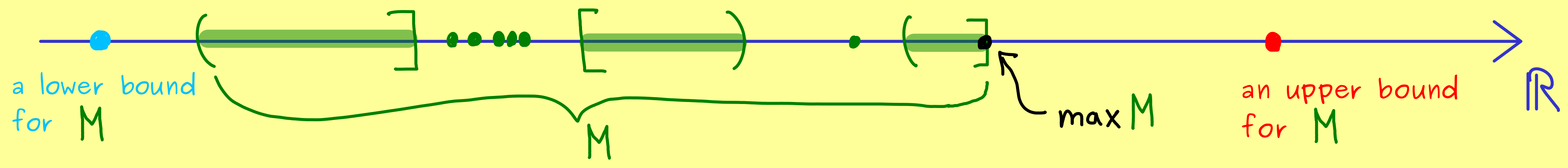


interval: $(a,b] := \{x \in \mathbb{R} \mid a < x \leq b\}$

$[a,b] := \{x \in \mathbb{R} \mid a \leq x \leq b\}$

$[a,\infty) := \{x \in \mathbb{R} \mid a \leq x\}$

$(-\infty,b) := \{x \in \mathbb{R} \mid x < b\}$



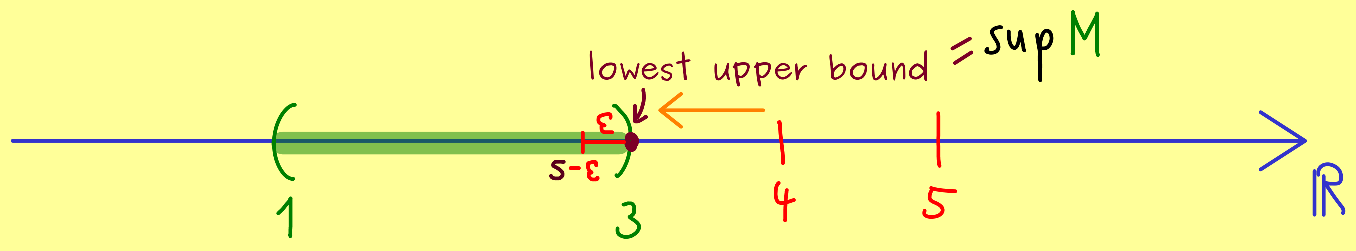
Definition: For a subset $M \subseteq \mathbb{R}$: $b \in \mathbb{R}$ is called <u>an upper bound for M</u> if $\forall x \in M: \; x \leq b$

$a \in \mathbb{R}$ is called <u>a lower bound for M</u> if $\forall x \in M: \; x \geq a$

If b is an upper bound for M <u>and</u> $b \in M$, then b is called a <u>maximal element of M</u>.

If a is a lower bound for M <u>and</u> $a \in M$, then a is called a <u>minimal element of M</u>.
$\min M$

Example:
- $M = [1,3]$, $\max M = 3$ $\quad \min M = 1$
- $M = (1,3)$, $\max M$, $\min M$ do not exist $\rightsquigarrow$ $\sup M$, $\inf M$



Definition: For a subset $M \subseteq \mathbb{R}$: $s \in \mathbb{R}$ is called <u>supremum of M</u> if
- $\forall x \in M: \; x \leq s$ (upper bound for M)
- $\forall \varepsilon > 0 \; \exists \tilde{x} \in M: \; s - \varepsilon < \tilde{x}$ ($s - \varepsilon$ is no upper bound for M)

Then write: $\sup M := s$ or $\sup M := \infty$ if M is not bounded from above
or $\sup \emptyset := -\infty$

For a subset $M \subseteq \mathbb{R}$: $\ell \in \mathbb{R}$ is called <u>infimum of M</u> if
- $\forall x \in M: \; x \geq \ell$ (lower bound for M)
- $\forall \varepsilon > 0 \; \exists \tilde{x} \in M: \; \ell + \varepsilon > \tilde{x}$ ($\ell + \varepsilon$ is no lower bound for M)

Then write: $\inf M := \ell$ or $\inf M := -\infty$ if M is not bounded from below
or $\inf \emptyset := \infty$