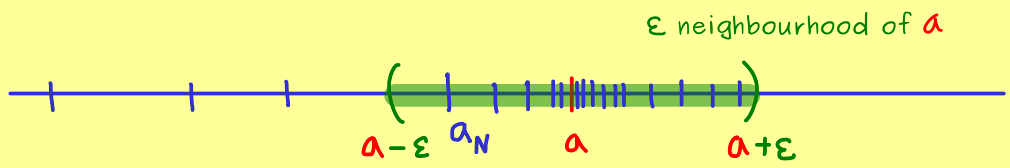
BECOME A MEMBER

ON STEADY

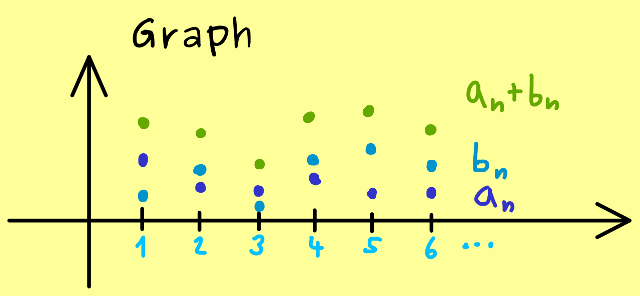
The Bright Side of Mathematics

Real Analysis – Part 4

$(a_n)_{n\in\mathbb{N}}$ convergent to $a \in \mathbb{R}$: $\lim\limits_{n\to\infty} a_n = a$



$a_n \xrightarrow{n\to\infty} a$



Theorem on limits: $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$ convergent sequences.

Then: (a)
$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

(b)
$$\lim_{n\to\infty}(a_n \cdot b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$$

(c)
$$\lim_{n\to\infty}\left(\frac{a_n}{\underbrace{b_n}_{\neq 0}}\right) = \frac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n} \neq 0$$

Example: $c_n = \dfrac{2n^2 + 5n - 1}{-5n^2 + n + 1}$ convergent? limit?

We know: $\frac{1}{n} \xrightarrow{n\to\infty} 0$

By (b): $\frac{1}{n}\cdot\frac{1}{n} \xrightarrow{n\to\infty} 0$

$$= \frac{\frac{1}{n^2}}{\frac{1}{n^2}} \cdot \frac{2n^2 + 5n - 1}{-5n^2 + n + 1} = \frac{2 + \frac{5}{n} - \frac{1}{n^2}}{-5 + \frac{1}{n} + \frac{1}{n^2}} \xrightarrow[\text{with limit theorems}]{n\to\infty} \frac{2+0-0}{-5+0+0} = -\frac{2}{5}$$