Exercise 1. Reordering a Series

We consider the alternating harmonic series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}$ *and denote the sum of the first* n *terms by* s_n*. The partial sums of the harmonic series* $\sum_{k=1}^{\infty} \frac{1}{k}$ *shall be denoted by* h_n*. Further we consider a reordering of the alternating harmonic series given by*

$$\frac{1}{1} + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \cdots .$$

The partial sums of this reordered series shall be denoted by t_n*. In this exercise, we will see that this reordering changes the limit of the partial sums.*

1. *Show that* $t_{3n} + \frac{1}{2}h_n = h_{4n} - \frac{1}{2}h_{2n}$ *for all* $n \in \mathbb{N}$*.*
2. *Deduce further*

$$3 \cdot \lim_{n\to\infty} s_n = 2 \cdot \lim_{n\to\infty} t_n$$

and explain why this implies the convergence of (t_n)*. What is the limit?*