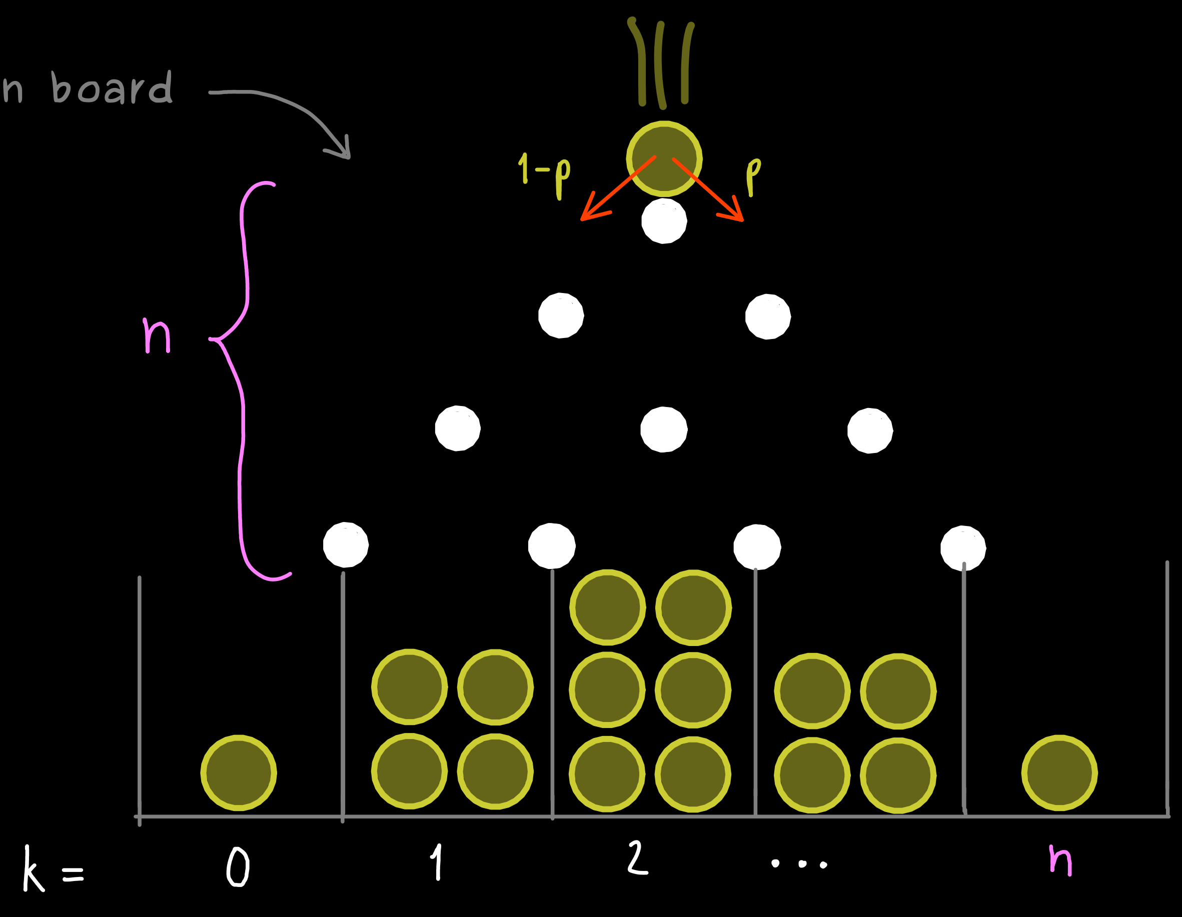
Probability Theory – Part 32

De Moivre-Laplace theorem (special case of central limit theorem)

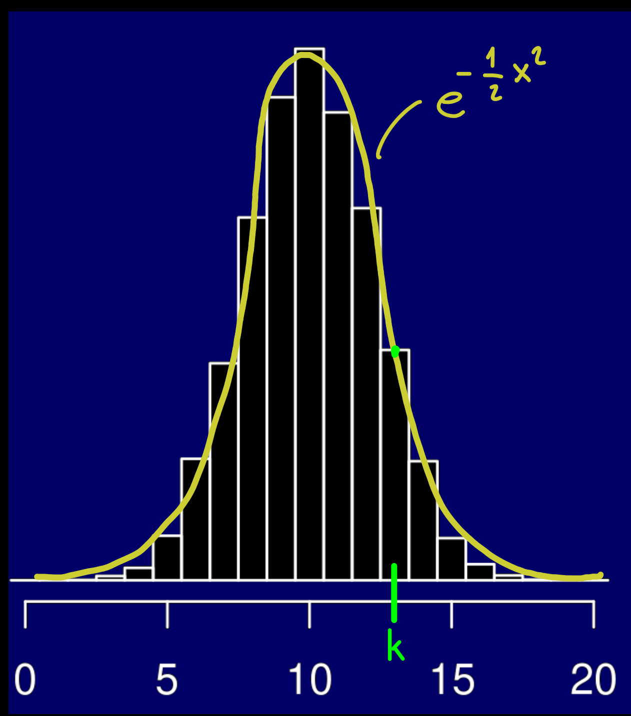
→ approximation of binomial distribution $\text{Bin}(n, p)$

→ n times Bernoulli (p)

Galton board



$\binom{n}{k} p^k (1-p)^{n-k}$

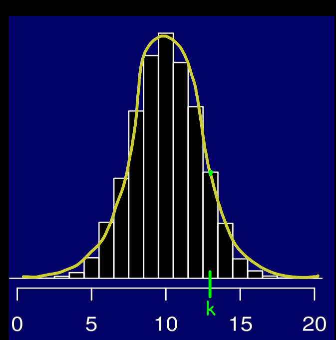


Result: For large n and k close to np, we get:

$$\binom{n}{k} p^k (1-p)^{n-k} \approx \frac{1}{\sqrt{2\pi n p(1-p)}} e^{-\frac{(k-np)^2}{2np(1-p)}}$$

pmf of binomial distribution

pdf of normal distribution



De Moivre-Laplace theorem: Let $p \in (0,1)$. Then for any $C > 0$:

$$\max_{\substack{0 \leq k \leq n \\ \text{with} \\ \frac{(k-np)^2}{2np(1-p)} \leq C}} \left| \frac{\sqrt{np(1-p)} \binom{n}{k} p^k (1-p)^{n-k}}{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(k-np)^2}{2np(1-p)}}} - 1 \right| \xrightarrow{n \to \infty} 0$$