The Bright Side of Mathematics

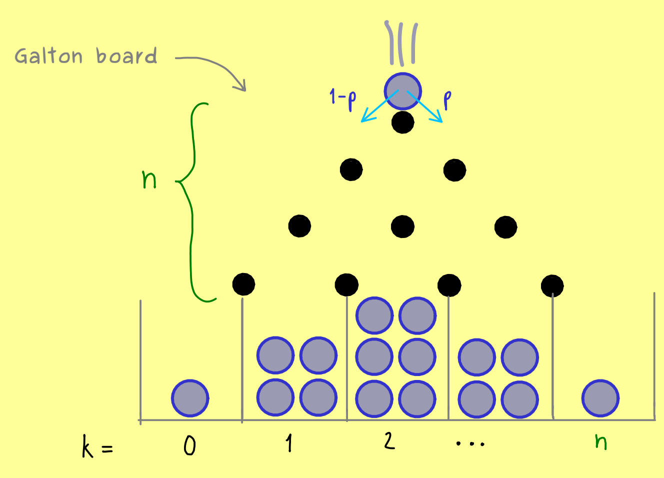
# Probability Theory - Part 32

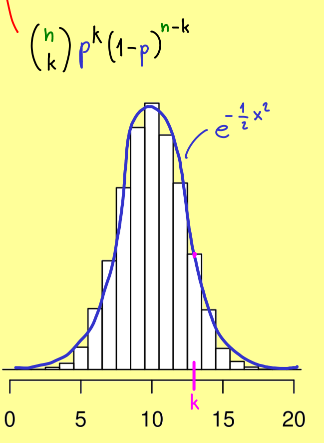
**De Moivre-Laplace theorem** (special case of central limit theorem)

→ approximation of binomial distribution $\text{Bin}(n, p)$

→ $n$ times Bernoulli $(p)$

$\binom{n}{k} p^k (1-p)^{n-k}$

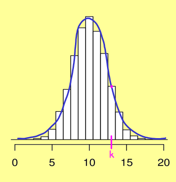



**Result:** For large $n$ and $k$ close to $np$, we get:

$$\binom{n}{k} p^k (1-p)^{n-k} \approx \frac{1}{\sqrt{2\pi n p (1-p)}} e^{-\frac{(k-np)^2}{2np(1-p)}}$$

pmf of binomial distribution (left side) — pdf of normal distribution (right side)




**De Moivre-Laplace theorem:** Let $p \in (0,1)$. Then for any $C > 0$:

$$\max_{\substack{0 \leq k \leq n \\ \text{with} \\ \frac{(k-np)^2}{2np(1-p)} \leq C}} \left| \frac{\sqrt{np(1-p)} \binom{n}{k} p^k (1-p)^{n-k}}{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(k-np)^2}{2np(1-p)}}} - 1 \right| \xrightarrow{n \to \infty} 0$$