The Bright Side of Mathematics

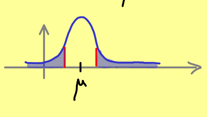
Probability Theory - Part 30

repeating a random experiment: $X_1, X_2, \ldots$ i.i.d. , $\mu := \mathbb{E}(X_1)$

should lead to: $\frac{1}{n}\sum_{k=1}^{n} X_k =: \overline{X}_n \xrightarrow{n\to\infty} \mu$

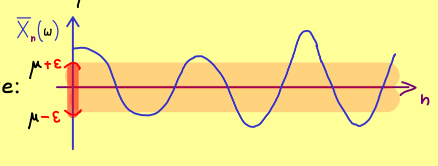
weak law of large numbers: $\left|\overline{X}_n(\omega) - \mu\right| \geq \varepsilon$ is unlikely for large n

$$\mathbb{P}\left(\left\{\omega\in\Omega \;\middle|\; \left|\overline{X}_n(\omega) - \mu\right| \geq \varepsilon\right\}\right) \xrightarrow{n\to\infty} 0$$



pointwise convergence? $\overline{X}_n(\omega) \xrightarrow{n\to\infty} \mu$?

we could have:



How many $\omega\in\Omega$ have such "bad" behaviour?

Strong law of large numbers: $X_k : \Omega \to \mathbb{R}$ random variables.

Let $(X_k)_{k\in\mathbb{N}}$ be i.i.d. and $\mathbb{E}(|X_1|) < \infty$.

Then for $\mu := \mathbb{E}(X_1)$: $\frac{1}{n}\sum_{k=1}^{n} X_k(\omega) =: \overline{X}_n(\omega) \xrightarrow{n\to\infty} \mu$ for $\omega\in\Omega$ almost surely

This means: $\mathbb{P}\left(\left\{\omega\in\Omega \;\middle|\; \overline{X}_n(\omega) \xrightarrow{n\to\infty} \mu\right\}\right) = 1$

(we could have $\overline{X}_n(\omega) \overset{n\to\infty}{\not\longrightarrow} \mu$ but the probability is zero)

Remark: almost sure convergence $\Longrightarrow$ convergence in probability

strong law of large numbers $\Longrightarrow$ weak law of large numbers