

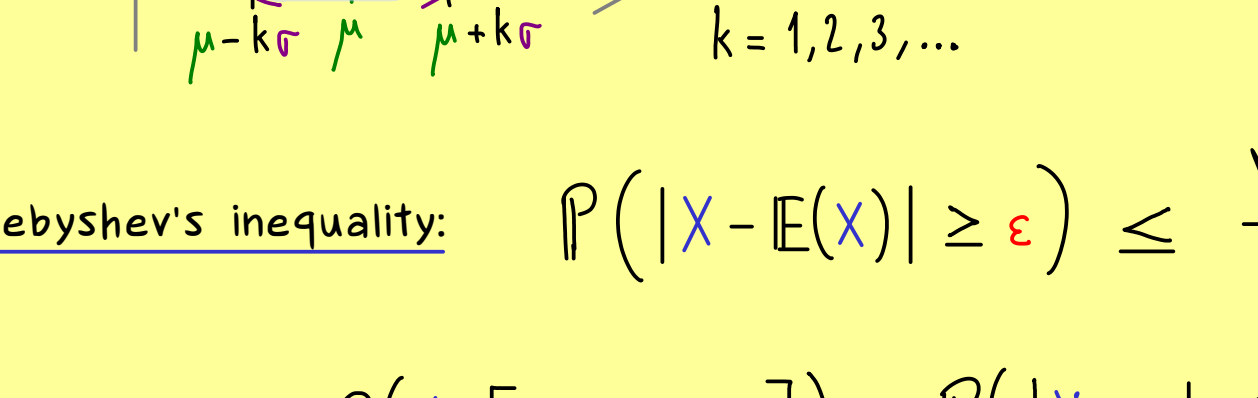


The Bright Side of Mathematics

Probability Theory - Part 27

Assumption: $X: \Omega \rightarrow \mathbb{R}$ random variable with

$$\begin{aligned} \mu &:= \mathbb{E}(X) \\ \sigma &:= \sqrt{\text{Var}(X)} \end{aligned} \quad \leftarrow \text{both should exist!}$$



Chebyshev's inequality: $\mathbb{P}(|X - \mathbb{E}(X)| \geq \epsilon) \leq \frac{\text{Var}(X)}{\epsilon^2}$

$k\sigma$ -intervals:

$$\begin{aligned} \mathbb{P}(X \in [\mu - k\sigma, \mu + k\sigma]) &= \mathbb{P}(|X - \mu| \leq k\sigma) \\ &\geq \mathbb{P}(|X - \mu| < k\sigma) \\ &= 1 - \mathbb{P}(|X - \mu| \geq k\sigma) \\ \text{Chebyshev's inequality} &\Rightarrow \geq 1 - \frac{\text{Var}(X)}{k^2\sigma^2} = 1 - \frac{1}{k^2} \end{aligned}$$

For $k=2$: $\mathbb{P}(X \in [\mu - 2\sigma, \mu + 2\sigma]) \geq 75\%$

For $k=3$: $\mathbb{P}(X \in [\mu - 3\sigma, \mu + 3\sigma]) \geq \frac{8}{9} \geq 88.8\%$

$k\sigma$ -intervals for the normal distribution: $\mu = 0, \sigma = 1$

$$\mathbb{P}(X \in [\mu - 1\sigma, \mu + 1\sigma])$$

$$\approx 0.682...$$

$$\mathbb{P}(X \in [\mu - 2\sigma, \mu + 2\sigma])$$

$$\approx 0.954...$$

$$\mathbb{P}(X \in [\mu - 3\sigma, \mu + 3\sigma])$$

$$\approx 0.997...$$

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1 n = 1000000
2 x = rnorm(n,0,1)
3 a = x[x >= -3 & x <= 3]
4 sigma3 = length(a)/length(x)
5 print(sigma3)

> a = x[x >= -3 & x <= 3]
> sigma3 = length(a)/length(x)
> print(sigma3)
[1] 0.9972977
    
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