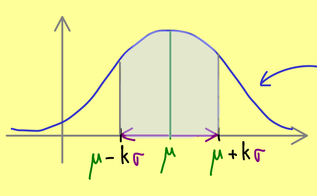
The Bright Side of Mathematics

Probability Theory - Part 27

Assumption: $X : \Omega \to \mathbb{R}$ random variable with

$$\mu := \mathbb{E}(X)$$
$$\sigma := \sqrt{\mathrm{Var}(X)}$$

both should exist!



probability density function

$k = 1,2,3,\dots$

Chebyshev's inequality:

$$\mathbb{P}\big(|X - \mathbb{E}(X)| \geq \varepsilon\big) \leq \frac{\mathrm{Var}(X)}{\varepsilon^2}$$

$k\sigma$-intervals:

$$\mathbb{P}\big(X \in [\mu - k\sigma, \mu + k\sigma]\big) = \mathbb{P}\big(|X - \mu| \leq k\sigma\big)$$
$$\geq \mathbb{P}\big(|X - \mu| < k\sigma\big)$$
$$= 1 - \mathbb{P}\big(|X - \mu| \geq k\sigma\big)$$

Chebyshev's inequality

$$\geq 1 - \frac{\mathrm{Var}(X)}{k^2 \sigma^2} = 1 - \frac{1}{k^2}$$

For $k = 2$: $\mathbb{P}\big(X \in [\mu - 2\sigma, \mu + 2\sigma]\big) \geq 75\%$

For $k = 3$: $\mathbb{P}\big(X \in [\mu - 3\sigma, \mu + 3\sigma]\big) \geq \frac{8}{9} \geq 88.8\%$

$k\sigma$-intervals for the normal distribution: $\mu = 0$, $\sigma = 1$

$$\mathbb{P}\big(X \in [\mu - 1\sigma, \mu + 1\sigma]\big) \approx 0.682\dots$$

$$\mathbb{P}\big(X \in [\mu - 2\sigma, \mu + 2\sigma]\big) \approx 0.954\dots$$

$$\mathbb{P}\big(X \in [\mu - 3\sigma, \mu + 3\sigma]\big) \approx 0.997\dots$$