BECOME A MEMBER

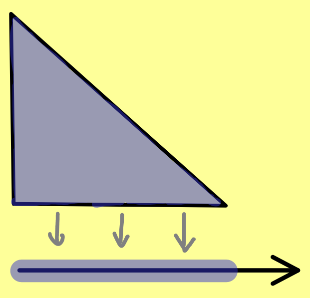
ON STEADY

The Bright Side of Mathematics

Probability Theory – Part 20

$X: \Omega \to \mathbb{R}^n$ random vector

$\Rightarrow \quad X_1: \Omega \to \mathbb{R}$ random variable



$$\begin{tikzcd}
(\Omega, \mathcal{A}, \mathbb{P}) \arrow[r, bend left, "X"] \arrow[rr, bend right, "X_1"'] & (\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n), \mathbb{P}_X) \arrow[r, bend left, "T"] & (\mathbb{R}, \mathcal{B}(\mathbb{R}), \cdot\,)
\end{tikzcd}$$

projection to first component

marginal distribution

Definition: $\mathbb{P}_{X_1} = (\mathbb{P}_X)_T$ is called the marginal distribution of X with respect to the first component.

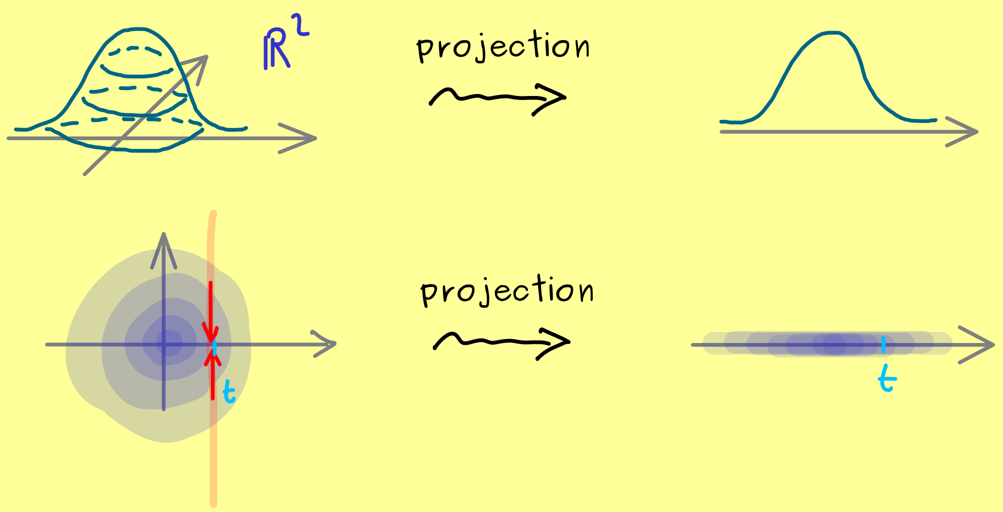
$$F_{X_1}(t) = \mathbb{P}_{X_1}\big((-\infty, t]\big) \qquad \text{marginal cumulative distribution function}$$

$$= \mathbb{P}_X\big((-\infty, t] \times \mathbb{R} \times \cdots \times \mathbb{R}\big)$$

$$= \mathbb{P}\big(X_1 \leq t,\ X_2 \in \mathbb{R}, \ldots, X_n \in \mathbb{R}\big)$$

Two important cases:

(1) (abs.) continuous: $\mathbb{P}_X$ has a probability density function $f_X: \mathbb{R}^n \to \mathbb{R}$



$$f_{X_1}(t) = \int_{\mathbb{R}^{n-1}} f_X(t, x_2, x_3, \ldots, x_n)\, d(x_2, \ldots, x_n) \qquad \text{marginal probability density function}$$

(2) discrete: $\mathbb{P}_X$ has a probability mass function $(p_x)_{x \in \mathbb{R}^n}$

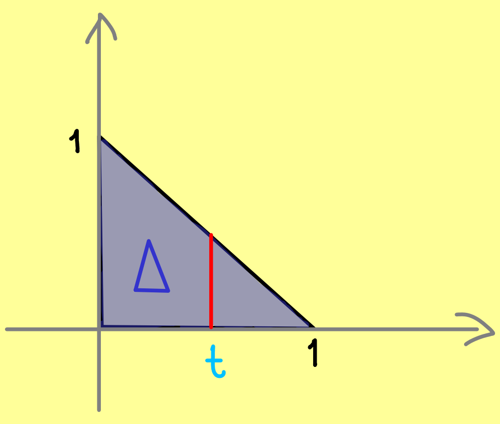
(only countably many are non-zero)

marginal probability mass function $(p_t)_{t \in \mathbb{R}}$ with

$$p_t = \sum_{x_2, x_3, \ldots \in \mathbb{R}} p(t, x_2, x_3, \ldots, x_n)$$

Example: $X: \Omega \to \mathbb{R}^2$ uniformly distributed on Δ

$$f_X(x_1, x_2) = \begin{cases} 2, & (x_1, x_2) \in \Delta \\ 0, & (x_1, x_2) \notin \Delta \end{cases}$$



marginal probability density function $f_{X_1}(t) = \int_{-\infty}^{\infty} f_X(t, x_2)\, dx_2$

$$= \begin{cases} \int_0^{1-t} 2\, dx_2 & t \in [0,1] \\ 0, & t \notin [0,1] \end{cases}$$

$$= \begin{cases} 2 - 2t, & t \in [0,1] \\ 0, & t \notin [0,1] \end{cases}$$