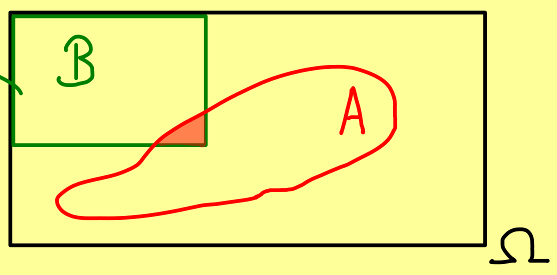
The Bright Side of Mathematics

## Probability Theory – Part 7

Conditional probability: $(\Omega, \mathcal{A}, \mathbb{P})$ probability space

subset $B \in \mathcal{A}$

with $\mathbb{P}(B) \neq 0$




$\Longrightarrow$ new probability space: $(B, \tilde{\mathcal{A}}, \tilde{\mathbb{P}})$ $\quad \tilde{\mathbb{P}}(A) = \dfrac{\mathbb{P}(A)}{\mathbb{P}(B)}$

only $A \in \mathcal{A}$ with $A \subseteq B$

$\Longrightarrow$ new probability space: $(\Omega, \mathcal{A}, \mathbb{P}_B)$ $\quad \mathbb{P}_B(A) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

Definition: $(\Omega, \mathcal{A}, \mathbb{P})$ probability space, $B \in \mathcal{A}$ with $\mathbb{P}(B) \neq 0$.

$\mathbb{P}(A \mid B) := \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$ is called the conditional probability of $A$ under $B$

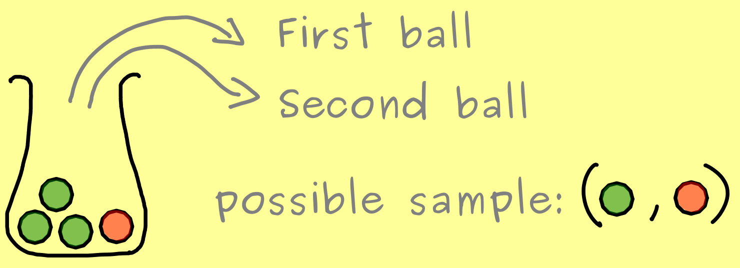
$\mathbb{P}(\cdot \mid B) : \mathcal{A} \longrightarrow [0,1]$ is called the conditional probability measure given $B$

Property: $\mathbb{P}(B \mid B) = 1$ $\qquad$ (For $\mathbb{P}(B) = 0$, set $\mathbb{P}(A \mid B) := 0$)

Example: urn model: ordered, without replacement

$C := \{g, r\}$, $\quad \Omega = C \times C$

$\mathcal{A} = \mathcal{P}(\Omega)$



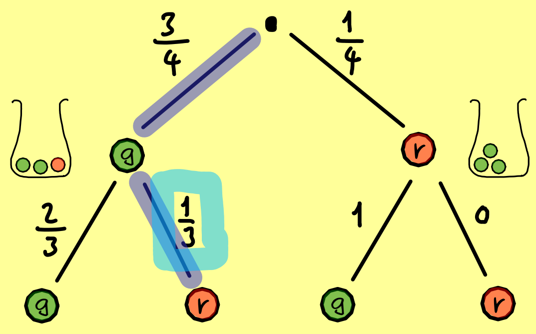

$\mathbb{P}$ given by probability mass function

$\mathbb{P}(\{(g,g)\}) = \frac{1}{2}$

$\mathbb{P}(\{(g,r)\}) = \frac{1}{4}$

$\mathbb{P}(\{(r,g)\}) = \frac{1}{4}$

$\mathbb{P}(\{(r,r)\}) = 0$




event: $B$ = "first ball is green" $= \{(g,g), (g,r)\}$

$$\mathbb{P}(\{(g,r)\} \mid B) = \frac{\mathbb{P}(\{(g,r)\} \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(\{(g,r)\})}{\mathbb{P}(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$$