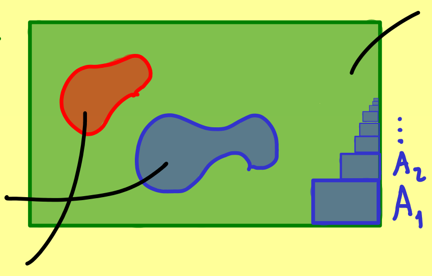
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Probability Theory – Part 2

Probability measures: measures with total mass $= 1$



$\mathbb{P}: \mathcal{A} \longrightarrow \mathbb{R}$ collection of subsets

We want:
- $\mathbb{P}(\Omega) = 1$, $\mathbb{P}(\emptyset) = 0$
- $\mathbb{P}(A) \in [0,1]$
- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ if A, B are disjoint $\hookrightarrow A \cap B = \emptyset$
- $\mathbb{P}\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} \mathbb{P}(A_j)$ if we have pairwise disjoint sets $\hookrightarrow A_i \cap A_j = \emptyset$ for $i \neq j$

Definition: Let Ω be a set. A collection of subsets $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ (power set) is called a sigma algebra if:

$\boxed{\sigma\text{-algebra}}$

(a) $\emptyset, \Omega \in \mathcal{A}$

(b) If $A \in \mathcal{A}$, then $A^c := \Omega \setminus A \in \mathcal{A}$

(c) If $A_1, A_2, \ldots \in \mathcal{A}$, then $\bigcup_{j=1}^{\infty} A_j \in \mathcal{A}$

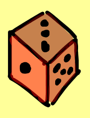
elements $A \in \mathcal{A}$ are called events

Definition: Let $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ be a σ-algebra. A map $\mathbb{P}: \mathcal{A} \longrightarrow [0,1]$ is called a probability measure if:

(a) $\mathbb{P}(\Omega) = 1$, $\mathbb{P}(\emptyset) = 0$

(b) $\mathbb{P}\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} \mathbb{P}(A_j)$

if we have pairwise disjoint sets $(A_i \cap A_j = \emptyset$ for $i \neq j)$

Example:  1 throw: $\Omega = \{1,2,3,4,5,6\}$

$\mathcal{A} = \mathcal{P}(\Omega)$

$\mathbb{P}: \mathcal{A} \longrightarrow [0,1]$, $\mathbb{P}(A) := \dfrac{\#A}{\#\Omega}$ (number of elements in a set)

For example: $\mathbb{P}(\{2\}) = \frac{1}{6}$, $\mathbb{P}(\{2,4,6\}) = \frac{3}{6} = \frac{1}{2}$

Exercise: Prove: $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$