Marginal distributions

Exercise 1. Marginals for Random Vectors

Let $X = (X_1, X_2)$ be a random vector with probability density function $f_X \colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f_X(s_1, s_2) := C \cdot \cos\left((|s_1| + |s_2|) \cdot \frac{\pi}{2}\right) \cdot \mathbf{1}_A(s_1, s_2)\,.$$

Here $A := \{(s_1, s_2) \in \mathbb{R}^2 \colon |s_1| + |s_2| \leq 1, s_1 \geq 0\}$ and C is a suitable constant.

(a) Draw the region A.

(b) Determine the correct value of C.

(c) Calculate the marginal densities f_{X_1}, f_{X_2}, i.e. the probability densities corresponding to the marginal distributions of X w.r.t. $k = 1, 2$.

(d) Calculate $\mathbb{E}(X_1)$ and $\mathbb{E}(X_2)$.

(e) Calculate $\mathbb{E}(X_1 \cdot X_2)$. Hint: Note that with $g \colon \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x \cdot y$ we have that $\mathbb{E}(X_1 \cdot X_2) = \mathbb{E}(g(X))$. Now use the change-of-variables formula, Proposition 3.31.

(f) Calculate $\mathrm{Cov}(X_1, X_2)$.