

## Covariance

### Exercise 1. Properties of Covariance

Let  $X$  and  $Y$  be random variables such that  $X^2$  and  $Y^2$  are integrable. Now let  $\text{Var}(X) = 1$ , show that the quadratic deviation

$$\mathbb{E}\left((Y - (a + bX))^2\right) \quad (*)$$

between  $Y$  and the affine function  $a + bX$  is minimized for  $b = \text{Cov}(X, Y)$  and  $a = \mathbb{E}(Y - bX)$ . What does this mean for the case that  $X$  and  $Y$  are uncorrelated?  
Hint: Resolve (\*) and take the partial derivatives with respect to  $a$  and  $b$ .