Properties of Variance

Exercise 1. Transformation of the Variance

Let X and Y be independent random variables with known variances:

$$Var(X) = 5, \quad Var(Y) = 3.$$

Find the variance of:

(a) $Z = 4X$

(b) $W = 2X + 3Y$

Exercise 2. Properties of Variance

Let X and Y be random variables such that X^2 and Y^2 are integrable.

(a) Using the properties you know, show that

$$\mathbb{E}((X - \mathbb{E}(X))^2) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

and

$$\mathbb{E}((X - \mathbb{E}(X)) \cdot (Y - \mathbb{E}(Y))) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y).$$

(b) Show that the expectation $\mathbb{E}(X)$ minimizes the quadratic deviation, i.e. for all $a \in \mathbb{R}$ we have

$$\mathbb{E}((X - a)^2) \geq \mathrm{Var}(X)$$

with equality if and only if $a = \mathbb{E}(X)$.