

Variance

Exercise 1. Some calculations for the variance

(a) Let $X: \Omega \rightarrow \mathbb{R}$ be a constant random variable, i.e. $X(\omega) = c$ for all $\omega \in \Omega$. Show that

$$\text{Var}(X) = 0.$$

(b) Let X be as in a and $Y: \Omega \rightarrow \mathbb{R}$ another random variable. Show that X and Y are independent and prove the identity $\text{Var}(X + Y) = \text{Var}(Y)$.

(c) Let X be a random variable such that X^2 is integrable. Consider the random variable

$$X^* := \frac{X - \mathbb{E}(X)}{\sqrt{\text{Var}(X)}}.$$

Use properties of expectation and variance to show that

$$\mathbb{E}(X^*) = 0 \quad \text{and} \quad \text{Var}(X^*) = 1.$$

Note: X^* is called *standardization* of X .