

Expectation

Exercise 1. Properties of the expectation

(a) Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and $A \in \mathcal{A}$. Show that $\mathbb{E}(\mathbf{1}_A) = \mathbb{P}(A)$. In particular, we have that $\mathbb{E}(\mathbf{1}_\Omega) = 1$.

Hint: By definition we have $\int \mathbf{1}_A d\mathbb{P} = \mathbb{P}(A)$.

(b) Let $X: \Omega \rightarrow \mathbb{R}$ be a constant random variable, i.e. $X(\omega) = c$ for all $\omega \in \Omega$. Show that

$$\mathbb{E}(X) = c$$

Note: In particular, for a random variable $Y: \Omega \rightarrow \mathbb{R}$ we have that $\mathbb{E}(\mathbb{E}(Y)) = \mathbb{E}(Y)$.