Cumulative Distribution Function

Exercise 1. Cumulative Distribution Functions and Probability Densities

1. *Let $X, Y : \Omega \to \mathbb{R}$ be random variables with $X \sim \textsc{Normal}(0, 1^2)$ and $Y \sim \textsc{Uniform}[0, 1]$.*

   Determine the CDF and the PDF of the following random variables:

   a) $X_1 := \sigma X + \mu$ *for fixed* $\sigma > 0$ *and* $\mu \in \mathbb{R}$

   b) $X_2 := |X|$

   c) $Y_1 := -Y + c$ *for fixed* $c > 0$

   d) $Y_2 := -\frac{1}{\lambda} \ln(1 - Y)$ *for fixed* $\lambda > 0$.

   Note: $Z := \alpha X + |Y|$ means: For each $\omega \in \Omega$ we set $Z(\omega) := \alpha X(\omega) + |Y(\omega)|$.

   Hint for (a): Recall that $\{X_1 \leq t\} = \{\sigma X + \mu \leq t\}$. Now take the probability of these events.

2. *Let* $X \sim \textsc{Normal}(\mu, \sigma^2)$, $a \neq 0$ *and* $b \in \mathbb{R}$. *Show that* $(aX + b) \sim \textsc{Normal}(a\mu + b, a^2\sigma^2)$.

3. *Let* $X \sim \textsc{Exp}(\lambda)$ *and* $a > 0$. *Show that* $aX \sim \textsc{Exp}(\lambda/a)$.