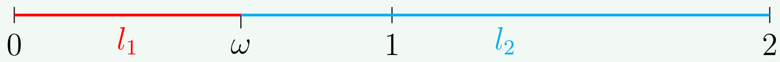
# Distribution of a Random Variable

## Exercise 1. Distribution Function of a Random Variable

*Consider the following random experiment: The interval* $[0,2]$ *will be divided into two parts by randomly (according to* $\textsc{Uniform}([0,1])$*) choosing one point* $\omega$ *of the interval* $\Omega := [0,1]$*. Let* $X\colon \Omega \to \mathbb{R}$ *be defined as the quotient* $l_1/l_2$ *of the shorter segment* $l_1$ *and the longer segment* $l_2$*.*




1. *Give a definition of* $X$ *in terms of* $\omega \in \Omega$*.*

2. *Determine the (cumulative) distribution function* $F_X$*. To this end, first show that*

$$\mathbb{P}(X \leq x) = \mathbb{P}\left(\left\{\omega \in \Omega \colon \omega \leq \frac{2x}{1+x}\right\}\right)$$

   *and then calculate* $F_X(x)$ *for* $x \in \mathbb{R}$*. You may distinguish the cases* $x < 0$*,* $0 \leq x \leq 1$ *and* $x > 1$*.*

3. *Based on your result from 2, calculate the probability density function* $f_X$ *corresponding to the probability distribution* $\mathbb{P}_X$*.*

## Exercise 2. Random Vectors

*Let* $(\Omega, \mathcal{A}, \mathbb{P})$ *be a probability space and* $X\colon \Omega \to \mathbb{R}^2$ *the random vector that describes (in polar coordinates) the location of a player's dart arrow when it hits a circular board with radius* $1$*. The distribution of* $X$ *is modelled via a probability density function* $f_X(r,\varphi) = r \cdot k \cdot \mathbf{1}_{[0,1]\times[0,2\pi)}(r,\varphi)$*.*

a) *Determine* $k$ *such that* $f_X$ *becomes a probability density function.*

b) *Give a set theoretical description of the following events and calculate their probability:*

   i. *The arrow hits the center of the board.*

   ii. *The arrow hits the board in the first quadrant.*

   iii. *The arrow misses the board.*

   iv. *The arrow hits the board in a distance of less than* $\frac{1}{2}$ *from the center.*

c) *Determine the cumulative distribution function* $F_X\colon \mathbb{R}^2 \to [0,1]$*.*