The Bright Side of Mathematics

Probability Theory – Part 15

$$\mathbb{E}(X) := \int_\Omega X \, d\mathbb{P}$$

Example: $\underbrace{X \sim \operatorname{Exp}(\lambda)}$ (exponential distribution)

$\mathbb{P}_X(A) = \int_A \underbrace{f_X(x)}_{\text{pdf}} dx$, $\quad f_X(x) = \begin{cases} \lambda e^{-\lambda \cdot x} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$

$$\mathbb{E}(X) = \int_\Omega X \, d\mathbb{P} = \int_\mathbb{R} x \cdot f_X(x) \, dx = \int_0^\infty x \cdot \lambda e^{-\lambda \cdot x} dx = \frac{1}{\lambda}$$

Properties: $(\Omega, \mathcal{A}, \mathbb{P})$ probability space, $X, Y : \Omega \longrightarrow \mathbb{R}$ random variables, where $\mathbb{E}(X)$ and $\mathbb{E}(Y)$ exist.

(a) $\mathbb{E}(a \cdot X + b \cdot Y) = a \cdot \mathbb{E}(X) + b \cdot \mathbb{E}(Y)$ for all $a, b \in \mathbb{R}$

(b) If X, Y are independent, then: $\mathbb{E}(X \cdot Y) = \mathbb{E}(X) \cdot \mathbb{E}(Y)$

(c) If $\mathbb{P}_X = \mathbb{P}_Y$, then: $\mathbb{E}(X) = \mathbb{E}(Y)$

(d) If $X \leq Y$ almost surely $\mathbb{P}(\{\omega \in \Omega \mid X(\omega) \leq Y(\omega)\}) = 1$,

then: $\mathbb{E}(X) \leq \mathbb{E}(Y)$